

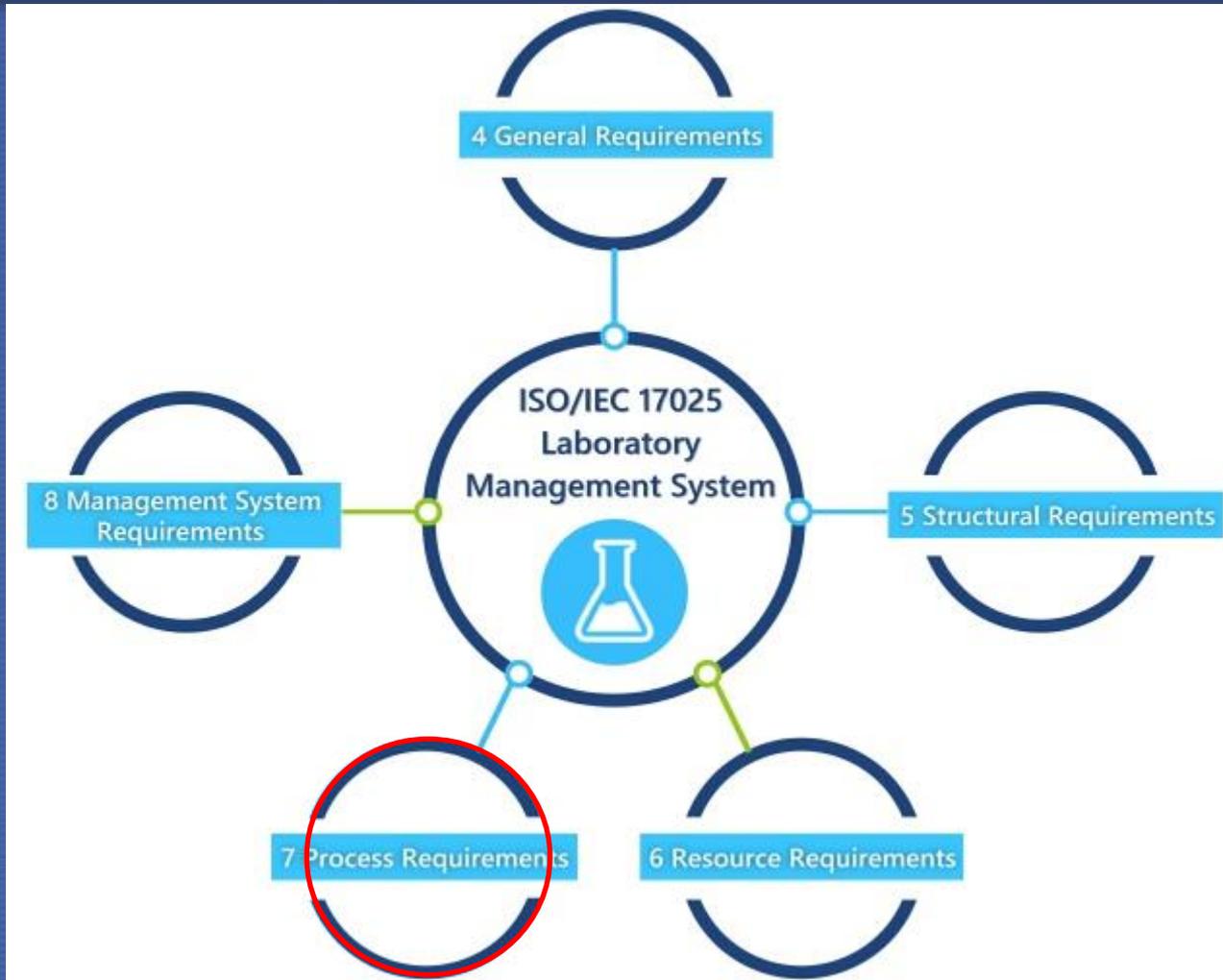
L16 Measurement Uncertainty



IAEA

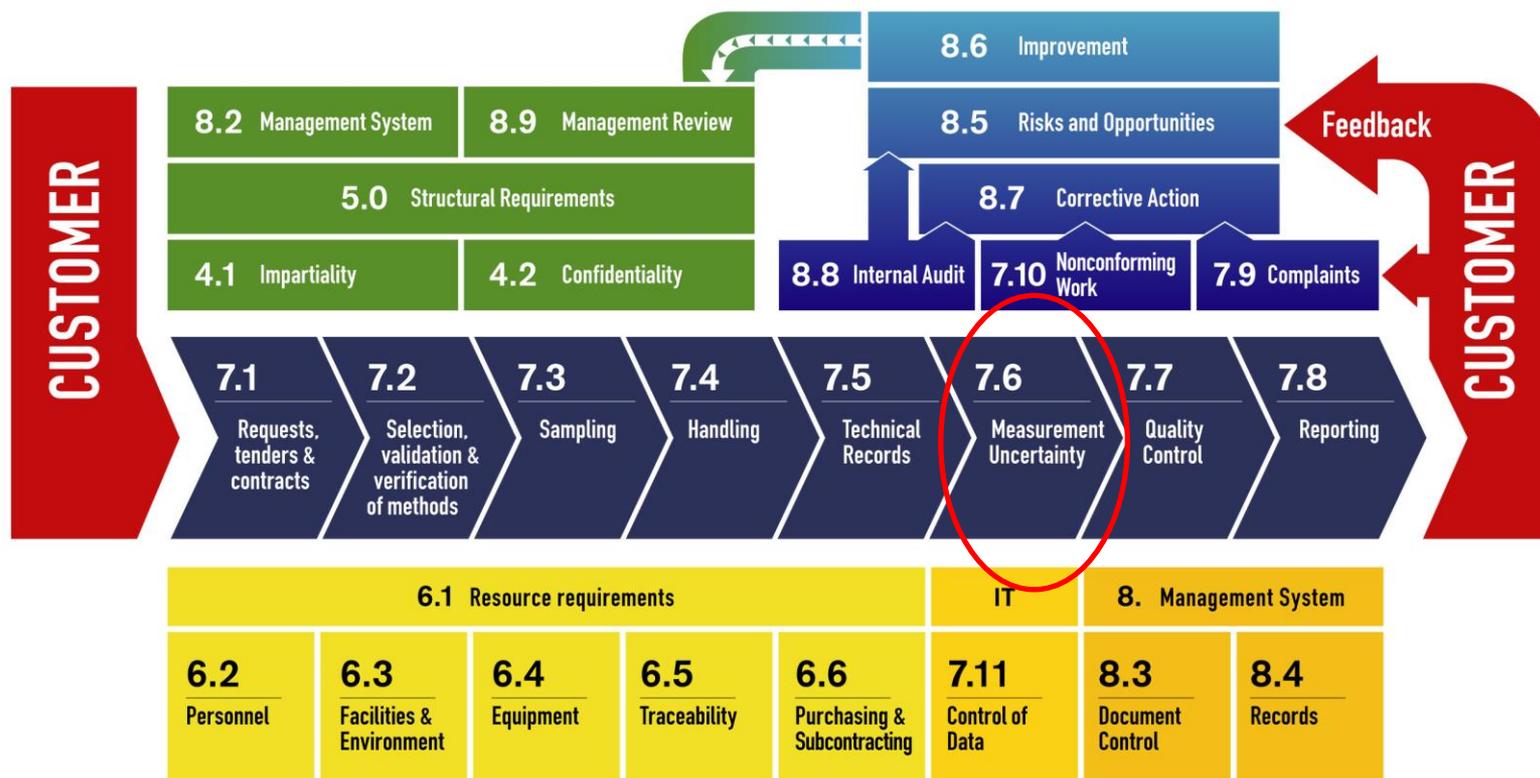
International Atomic Energy Agency

7 Process Requirements



Structure

ISO/IEC 17025: 2017



Objectives

In this lecture we will:

- Investigate measurement results
- Define measurement uncertainties and uncertainty budgets
- Investigate uncertainty contributions in dosimetry applications
- establish uncertainty budgets for all measurements, tests, and calibrations

Why?

- To ensure that clients benefit fully from laboratories' services, accredited testing laboratories have developed appropriate principles for their collaboration with clients. Clients have the right to expect that the test reports are factually correct, useful and comprehensive. Depending on the situation, clients are also interested in quality features, especially
 - the reliability of the results and a quantitative statement on this reliability, i.e. uncertainty
 - the level of confidence of a conformity statement about the product that can be inferred from the testing result and the associated expanded uncertainty”

7.6 Evaluation of measurement uncertainty

- 7.6.1 Identify all the contributions which are of significance (including sampling) to measurement uncertainty

- 7.6.2 For **calibration laboratories** it is expected to evaluate the measurement uncertainty for all calibrations considering all contributions which are of significance, including those arising from sampling.

7.6 Evaluation of measurement uncertainty (2)

7.6.3 For **testing laboratories** it is expected to evaluate measurement uncertainty considering all contributions which are of significance, including those arising from sampling

For testing laboratory evaluate measurement uncertainty. Where the test method precludes rigorous evaluation of measurement uncertainty, an estimation is made based on an understanding of the theoretical principles or practical experience of the performance of the method.

Recommendations for Uncertainty Budgets

- Investigation of all factors contributing to the total measurement uncertainty.
- Determination of the type and distribution of the uncertainty contribution.
- Determination of the size of the uncertainty contributions.
- Combination of the individual uncertainty contributions to obtain the total uncertainty = uncertainty budget.

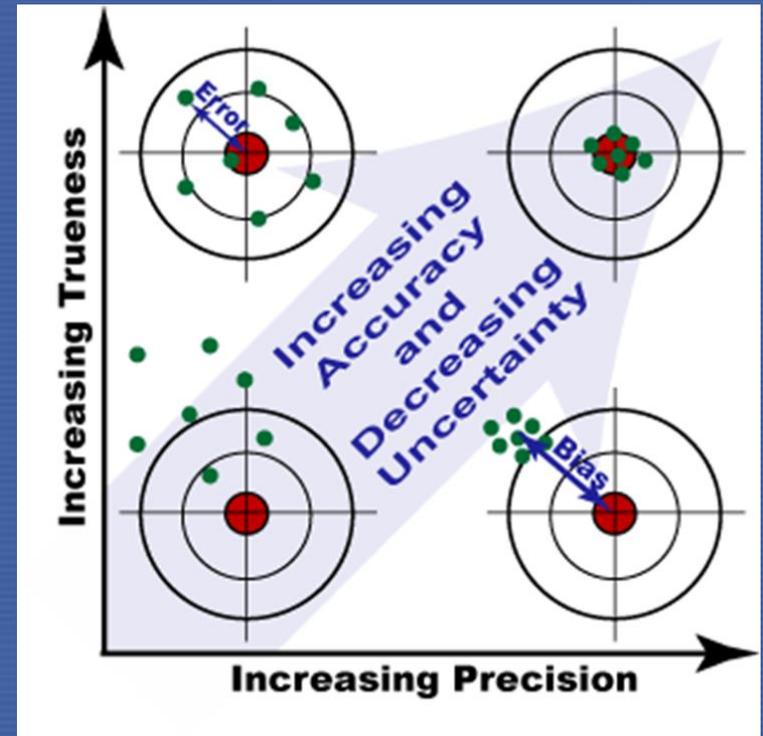
Measurement Result

- Shall be best estimate of measurand
- Supported by a combined uncertainty
- Taking into account
 - Type A uncertainties:
estimated from statistical considerations
 - Type B uncertainties:
obtained from other sources or references
- Expanded to a defined confidence level
(k-factor)

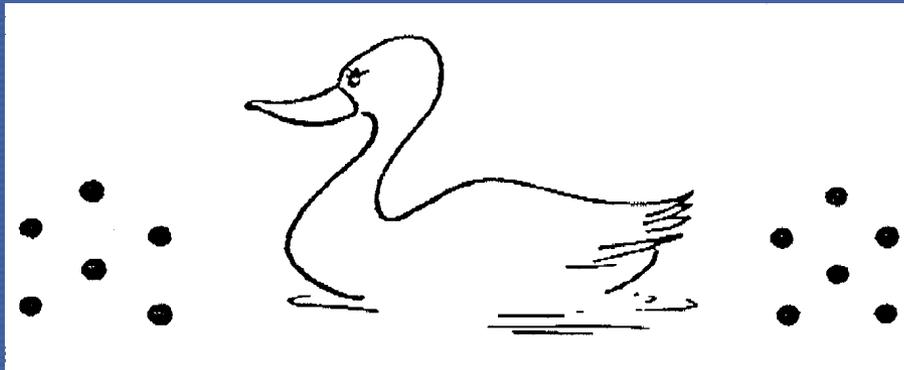


Measurement Uncertainty

- No measurement can be made with 100% confidence
- All measurements has influencing factors which can not be perfectly quantified.
- Hence all measurements statement shall include measurement uncertainty



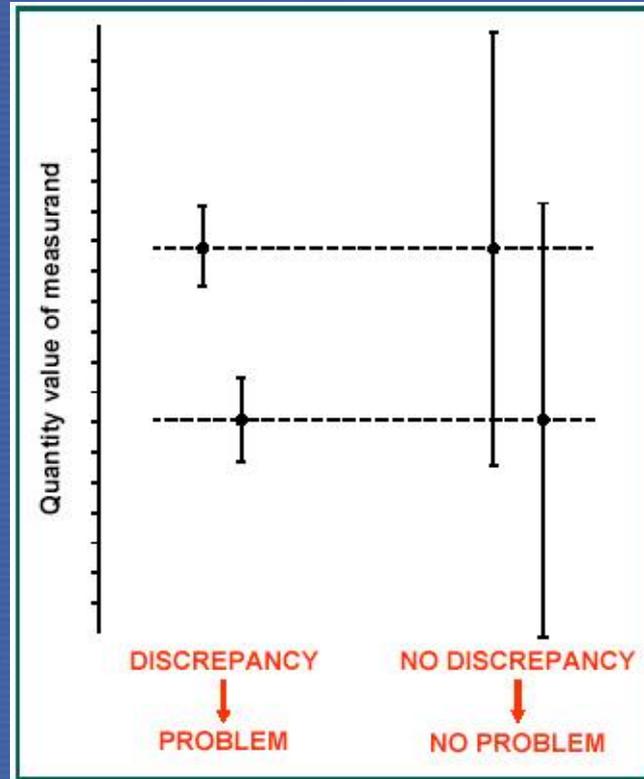
On the average the duck is dead



A hunter fired both barrels of a shotgun at a duck.
The first hit 50 cm in front, the second hit 50 cm behind.
On the average the duck was dead.
In duck hunting one wants a **single shot to hit the mark.**

It is cheaper to perform less measurements,
but have **sufficiently small uncertainty**,
meeting a pre-set “**target measurement uncertainty**”
than making many measurements and use the average

Can we interpret a result without its measurement uncertainty?



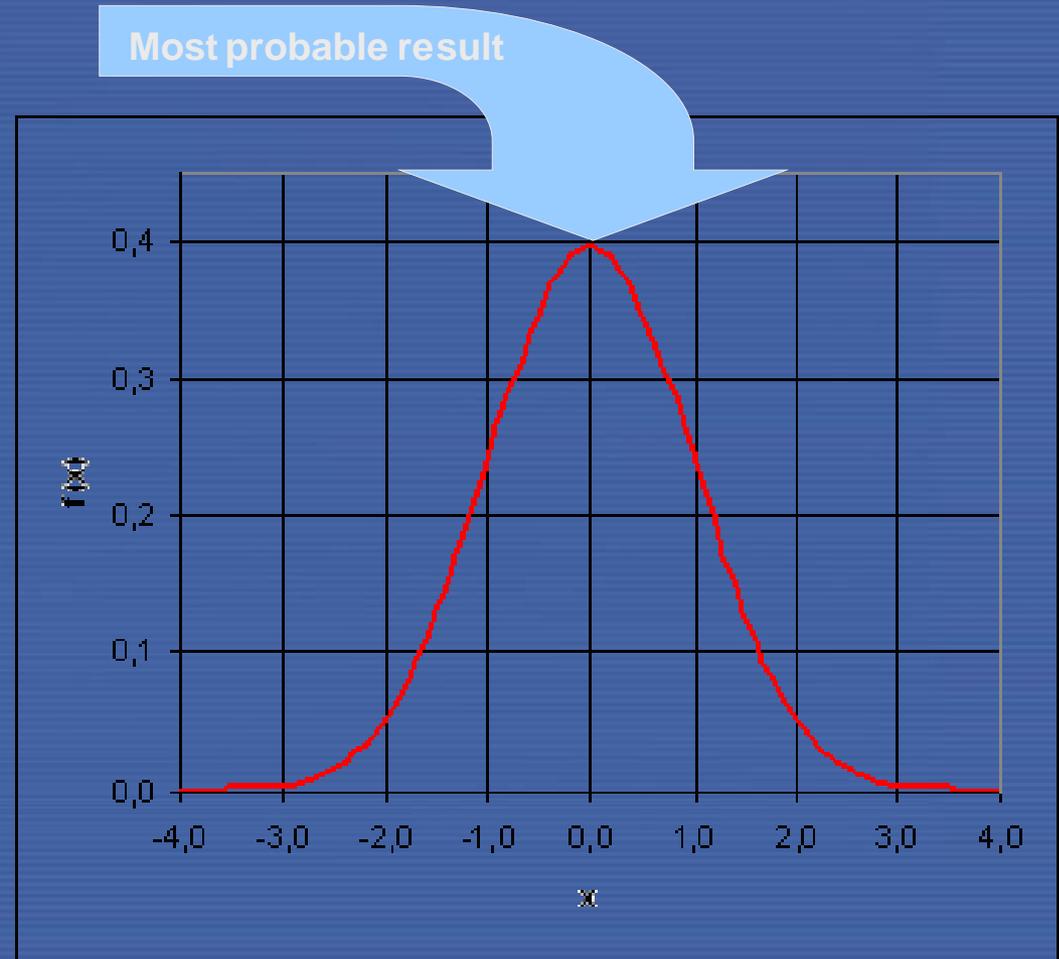
Only when the result is given with a measurement uncertainty stripped to all its important sources of uncertainty we can compare results

The two references



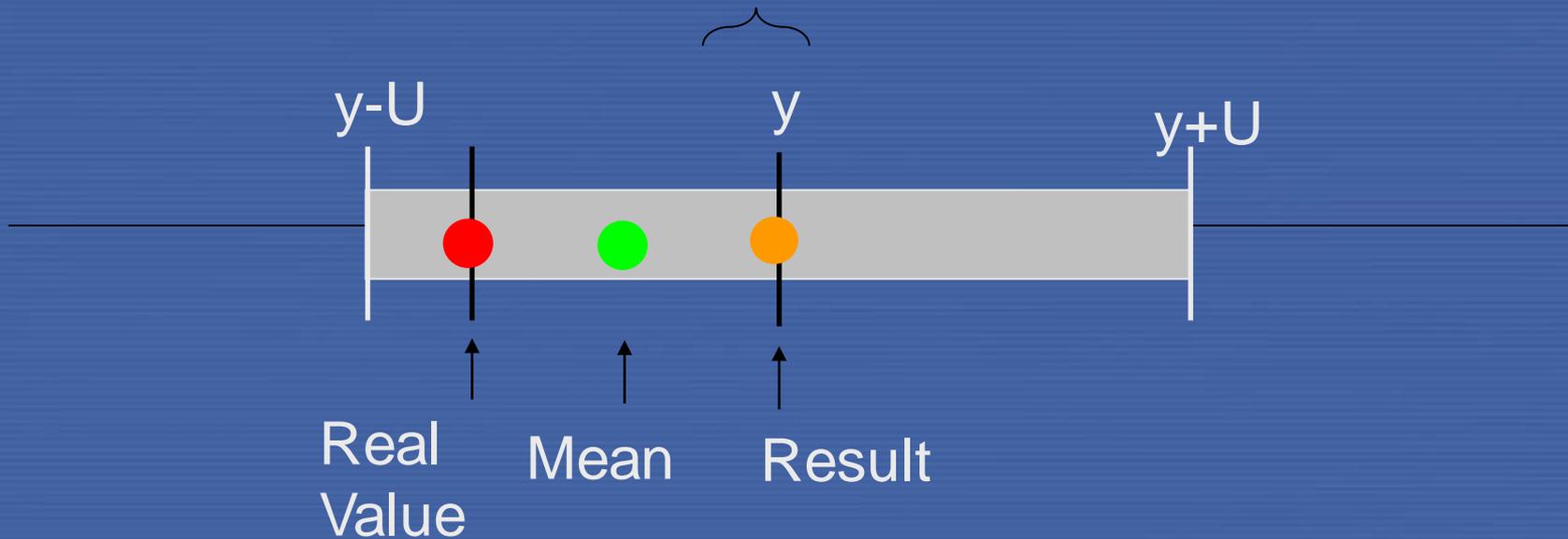
Best estimate of measurand

Due to random effects in measurement processes (short term fluctuations of influencing factors), we can never measure the exact quantity of a measurand.



Uncertainty

Uncertainty range $[y-U, y+U]$



There are no more certainties ...



Forget systematical error & random error

Type A or B measurement uncertainties

Advantage: each with its own calculation method

Definitions (VIM)

Measurement uncertainty

Non-negative parameter characterizing the dispersion of the **quantity values** being attributed to a **measurand**, based on the information used

- NOTE 1 Measurement uncertainty includes components arising from systematic effects, such as components associated with **corrections** and the assigned quantity values of **measurement standards**, as well as the **definitional uncertainty**. Sometimes estimated systematic effects are not corrected for but, instead, associated measurement uncertainty components are incorporated.
- NOTE 2 The parameter may be, for example, a standard deviation called **standard measurement uncertainty** (or a specified multiple of it), or the half-width of an interval, having a stated **coverage probability**.

Definitions (VIM)

- NOTE 3 Measurement uncertainty comprises, in general, many components. Some of these may be evaluated by **Type A evaluation of measurement uncertainty** from the statistical distribution of the quantity values from series of **measurements** and can be characterized by standard deviations. The other components, which may be evaluated by **Type B evaluation of measurement uncertainty**, can also be characterized by standard deviations, evaluated from probability density functions based on experience or other information.
- NOTE 4 In general, for a given set of information, it is understood that the measurement uncertainty is associated with a stated quantity value attributed to the measurand. A modification of this value results in a modification of the associated uncertainty.

Definitions (VIM)

Standard measurement uncertainty

Measurement uncertainty expressed as a standard deviation (u)

Combined standard measurement uncertainty (u_c)

Standard measurement uncertainty that is obtained using the individual **standard measurement uncertainties** associated with the **input quantities in a measurement model**

NOTE In case of correlations of input quantities in a measurement model, covariances must also be taken into account when calculating the combined standard measurement uncertainty; see also ISO/IEC Guide 98-3:2008

Definitions (VIM)

Relative standard measurement uncertainty

Standard measurement uncertainty divided by the absolute value of the measured quantity value

Uncertainty budget

Statement of a **measurement uncertainty**, of the components of that measurement uncertainty, and of their calculation and combination

NOTE An uncertainty budget should include the **measurement model**, estimates, and measurement uncertainties associated with the **quantities** in the measurement model, covariances, type of applied probability density functions, degrees of freedom, type of evaluation of measurement uncertainty, and any **coverage factor**.

Definitions (VIM)

Target measurement uncertainty (target uncertainty)

Measurement uncertainty specified as an upper limit and decided on the basis of the intended use of **measurement results**

Expanded measurement uncertainty (U)

Product of a **combined standard measurement uncertainty** and a factor larger than the number one

NOTE 1 The factor depends upon the type of probability distribution of the **output quantity in a measurement model** and on the selected **coverage probability**.

NOTE 2 The term “factor” in this definition refers to a **coverage factor**.

NOTE 3 Expanded measurement uncertainty is termed “overall uncertainty” in paragraph 5 of Recommendation INC-1 (1980) (see the GUM) and simply “uncertainty” in IEC documents.

Definitions (VIM)

Coverage interval

Interval containing the set of **true quantity values** of a **measurand** with a stated probability, based on the information available

NOTE 1 A coverage interval does not need to be centred on the chosen **measured quantity value** (see ISO/IEC Guide 98-3:2008/Suppl.1).

NOTE 2 A coverage interval should not be termed “confidence interval” to avoid confusion with the statistical concept (see ISO/IEC Guide 98-3:2008, 6.2.2).

NOTE 3 A coverage interval can be derived from an **expanded measurement uncertainty** (see ISO/IEC Guide 98-3:2008, 2.3.5).

Definitions (VIM)

Coverage probability

Probability that the set of **true quantity values** of a **measurand** is contained within a specified **coverage interval**

NOTE 1 This definition pertains to the Uncertainty Approach as presented in the GUM.

NOTE 2 The coverage probability is also termed “level of confidence” in the GUM.

Coverage factor

number larger than one by which a **combined standard measurement uncertainty** is multiplied to obtain an **expanded measurement uncertainty**

NOTE A coverage factor is usually symbolized k (see also ISO/IEC Guide 98-3:2008, 2.3.6).

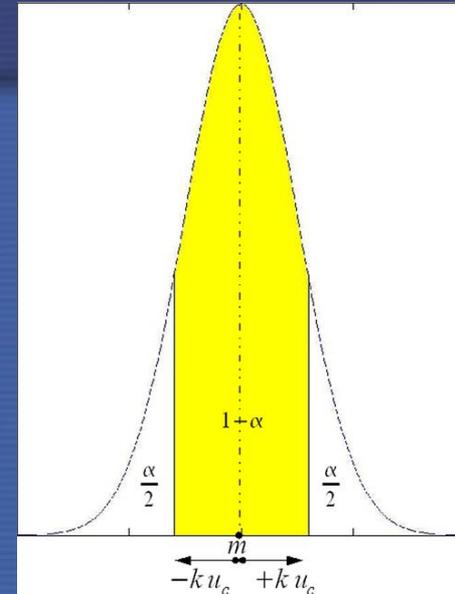
Expanded uncertainty

Confidence interval (if sufficient degrees of freedom):

99,7%: 3σ : $k=3$

95,5%: 2σ : $k=2$

$$U = k \cdot u_c$$



If a result is $(0,54 \pm 0,14)$ mSv ($k=2$) it means that the true dose has a 95 % probability of being between 0,40 and 0,68 mSv; but also that there is 5 % probability of being outside of this interval

Definitions VIM

Type A evaluation of measurement uncertainty - Type A evaluation: *evaluation of a component of **measurement uncertainty** by a statistical analysis of **measured quantity values** obtained under defined measurement conditions*

- NOTE 1 For various types of measurement conditions, see **repeatability condition of measurement, intermediate precision condition of measurement, and reproducibility condition of measurement.**

Uncertainty type A

Based on any valid statistical method for treating data.

Examples are:

- calculating the standard deviation of the mean of a series of independent observations;
- using the method of least squares to fit a curve to data in order to estimate the parameters of the curve and their standard deviations;
- carrying out an analysis of variance in order to identify and quantify random effects in certain kinds of measurements.

Type A uncertainty

Standard deviation (note that denominator is 1)

$$m = \frac{1}{n} \sum_{i=1}^n x_i ; s = \sqrt{\frac{1}{n-1} \sum_{i=1}^n (x_i - m)^2}$$

Standard error of the mean:

$$se_m = \frac{s}{\sqrt{n}}$$

Definitions VIM

Type B evaluation of measurement uncertainty - Type B evaluation: *evaluation of a component of **measurement uncertainty** determined by means other than a **Type A evaluation of measurement uncertainty***

EXAMPLES Evaluation based on information

- associated with authoritative published **quantity values**,
- associated with the quantity value of a **certified reference material**,
- obtained from a **calibration** certificate,
- about drift,
- obtained from the **accuracy class** of a verified **measuring instrument**,
- obtained from limits deduced through personal experience.

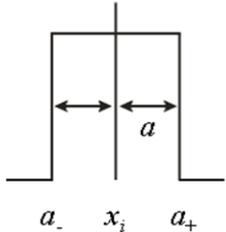
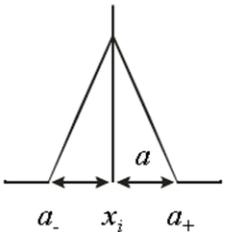
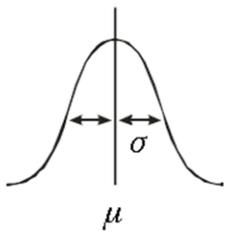
Uncertainty type B

Based on scientific judgment using all of the relevant information available, which may include:

- previous measurement data,
- experience with, or general knowledge of, the behaviour and property of relevant materials and instruments,
- manufacturer's specifications,
- data provided in calibration and other reports, and
- uncertainties assigned to reference data taken from handbooks.

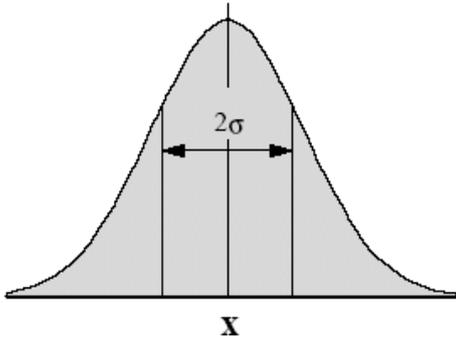
Type B uncertainty

most common pdfs

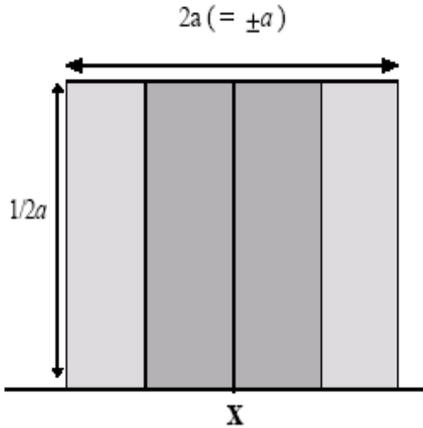
	rectangular	triangular	normal
pdf			
x_i	$(a_+ + a_-)/2$	$(a_+ + a_-)/2$	μ
$u(x_i)$	$a/\sqrt{3}$	$a/\sqrt{6}$	σ

In order to convert to standard uncertainty we need to divide by the pdf = probability density function

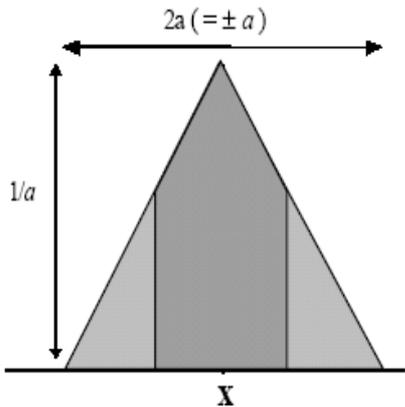
Normal distribution

Normal distribution		
Form	Use when:	Uncertainty
	<ul style="list-style-type: none"> An estimate is made from repeated observations of a randomly varying process. An uncertainty is given in the form of a standard deviation s, a relative standard deviation s/\bar{x}, or a coefficient of variance $CV\%$ without specifying the distribution. An uncertainty is given in the form of a 95% (or other) confidence interval $x \pm c$ without specifying the distribution. 	$u(x) = s$ $u(x) = s$ $u(x) = x \cdot (s / \bar{x})$ $u(x) = \frac{CV\%}{100} \cdot x$ $u(x) = c / 2$ (for c at 95%) $u(x) = c / 3$ (for c at 99.7%)

Rectangular distribution

Rectangular distribution		
Form	Use when:	Uncertainty
	<ul style="list-style-type: none">• A certificate or other specification gives limits without specifying a level of confidence (e.g. 25ml ± 0.05ml)• An estimate is made in the form of a maximum range ($\pm a$) with no knowledge of the shape of the distribution.	$u(x) = \frac{a}{\sqrt{3}}$

Triangular distribution

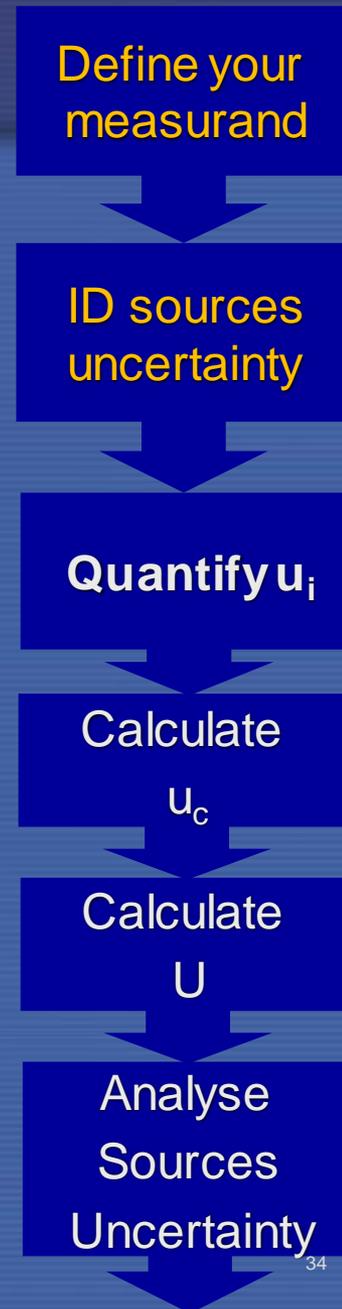
Triangular distribution		
Form	Use when:	Uncertainty
 <p>The diagram shows a gray-shaded triangle on a horizontal axis labeled 'X'. A double-headed arrow above the triangle indicates its base width is $2a (= \pm a)$. A vertical arrow to the left of the triangle indicates its height is $1/a$.</p>	<ul style="list-style-type: none">• The available information concerning x is less limited than for a rectangular distribution. Values close to x are more likely than near the bounds.• An estimate is made in the form of a maximum range ($\pm a$) described by a symmetric distribution.	$u(x) = \frac{a}{\sqrt{6}}$

Formulation stage

The formulation stage constitutes of:

- Defining the output quantity Y (the measurand, in our case a dose, for example $H_p(10)$).
- Determining the input and influence quantities $x = x_1, x_2, \dots, x_n$.
- These are all the quantities that affect the value of the output quantity, in our case the radiation field characteristics (for example dose rate, energy and angle of incidence), dosimeter characteristics (for example sensitivity as a function of radiation energy and angle of incidence, fading), characteristics of the evaluating system (for example developer temperature, linearity or TLD reader sensitivity) and characteristics of the calibration system.

An important input quantity, in particular for the low dose performance of a dosimeter, is the subtraction of the dose due to natural background radiation.



Formulation stage

- The formulation stage constitutes of:
 - Developing a model relating the input quantities to the output quantity $Y = f(X)$. In most cases the model is already largely available in the form of the algorithm that is routinely used to calculate the dose including factors such as calibration and normalization factors or coefficients, fading coefficients, instrumental blank and background dose.
 - Assigning a probability density function, PDF, to each of the input quantities x_i .
- This assignment is done using all knowledge of the dosimetry system and the measurement conditions.

Define your
measurand

ID sources
uncertainty

Quantify u_i

Calculate

u_c

Calculate

U

Analyse
Sources
Uncertainty

Formulation stage: example of an IMS

$$y = y_{gross} - tH_{bg}$$

$$y_{gross} = \frac{x - z}{f_{REF} f_{TLD} f_{E,\alpha}}$$

- x : is the reader signal from the detector
- z : the reader blank signal
- f_{TLD} : ECC (element correction coefficient)
- f_{REF} : RCF (reader calibration factor)
- $f_{E,\alpha}$: is the correction for the energy, angle, fading, linearity, light exposure, climatic, ...
- t days between annealing and readout
- H_{bg} local customer background in mSv/day

Define your
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Analyse
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Uncertainty

Formulation stage: example sources IMS

- u_x : type A (repeatability – counting statistics)
- u_{reader} : stability of the reader
- u_z : the reader blank signal
- $u_{f,\text{ECC}}$: Element correction coefficient + batch homogeneity of the detectors
- $u_{f,\text{REF}}$: Calibration uncertainty SSDL
- $u_{f,\text{E}}$: Energy dependence
- $u_{f,\alpha}$: Angular dependence
- $u_{f,\text{fading}}$: Fading
- $u_{f,\text{lin}}$: Linearity response
- $u_{f,\text{env}}$: Light exposure, climatic, ... effects
- $u_{\text{H,bg}}$: local customer background in mSv/day
- u_{field} : Nature of the field: photons, beta's, mix
- ...

Define your
measurand

ID sources
uncertainty

Quantify u_i

Calculate

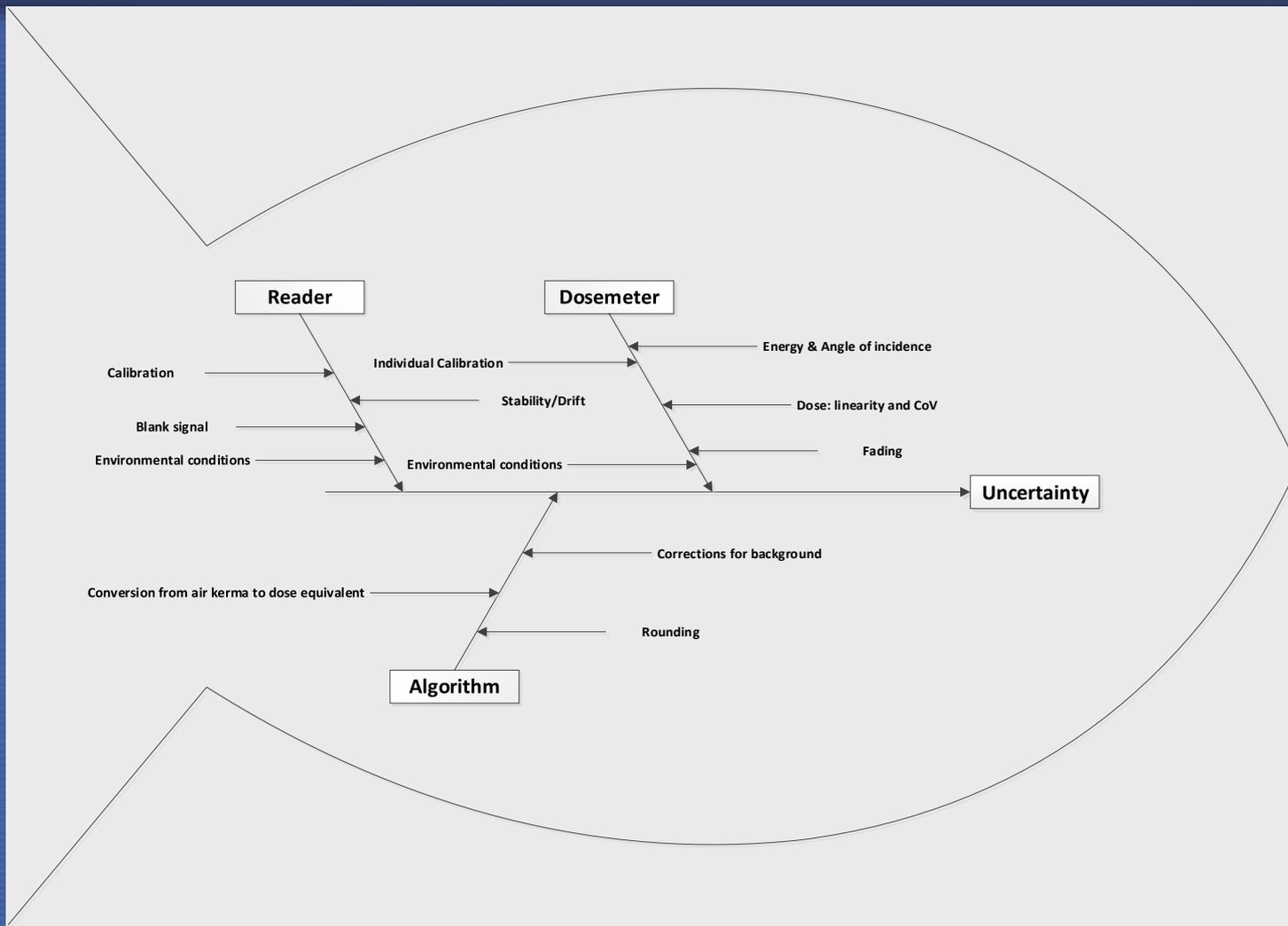
u_c

Calculate

U

Analyse
Sources
Uncertainty

Identify your sources of uncertainty – e.g. IEC 62461 or RP 160



Define your
measurand

ID sources
uncertainty

Quantify u_i

Calculate

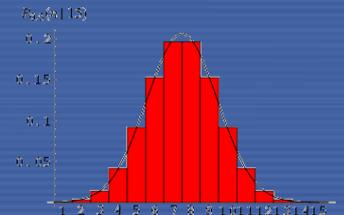
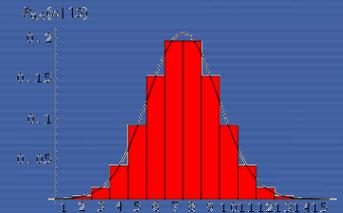
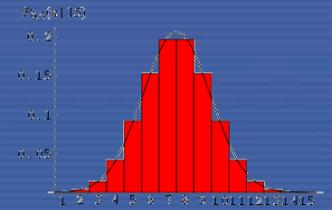
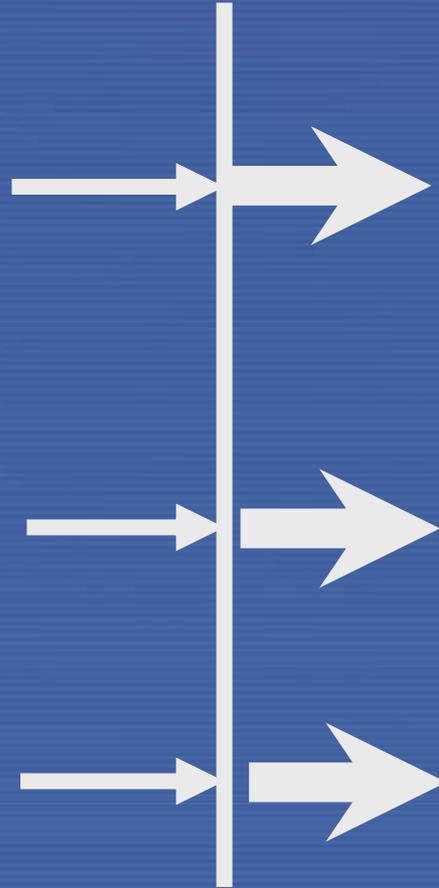
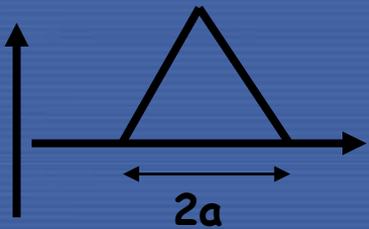
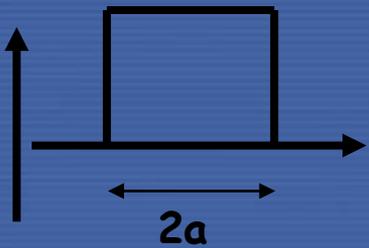
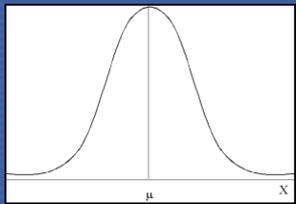
u_c

Calculate

U

Analyse
Sources
Uncertainty

Convert to standard uncertainties using pdf



Define your measurand

ID sources uncertainty

Quantify u_i

Calculate

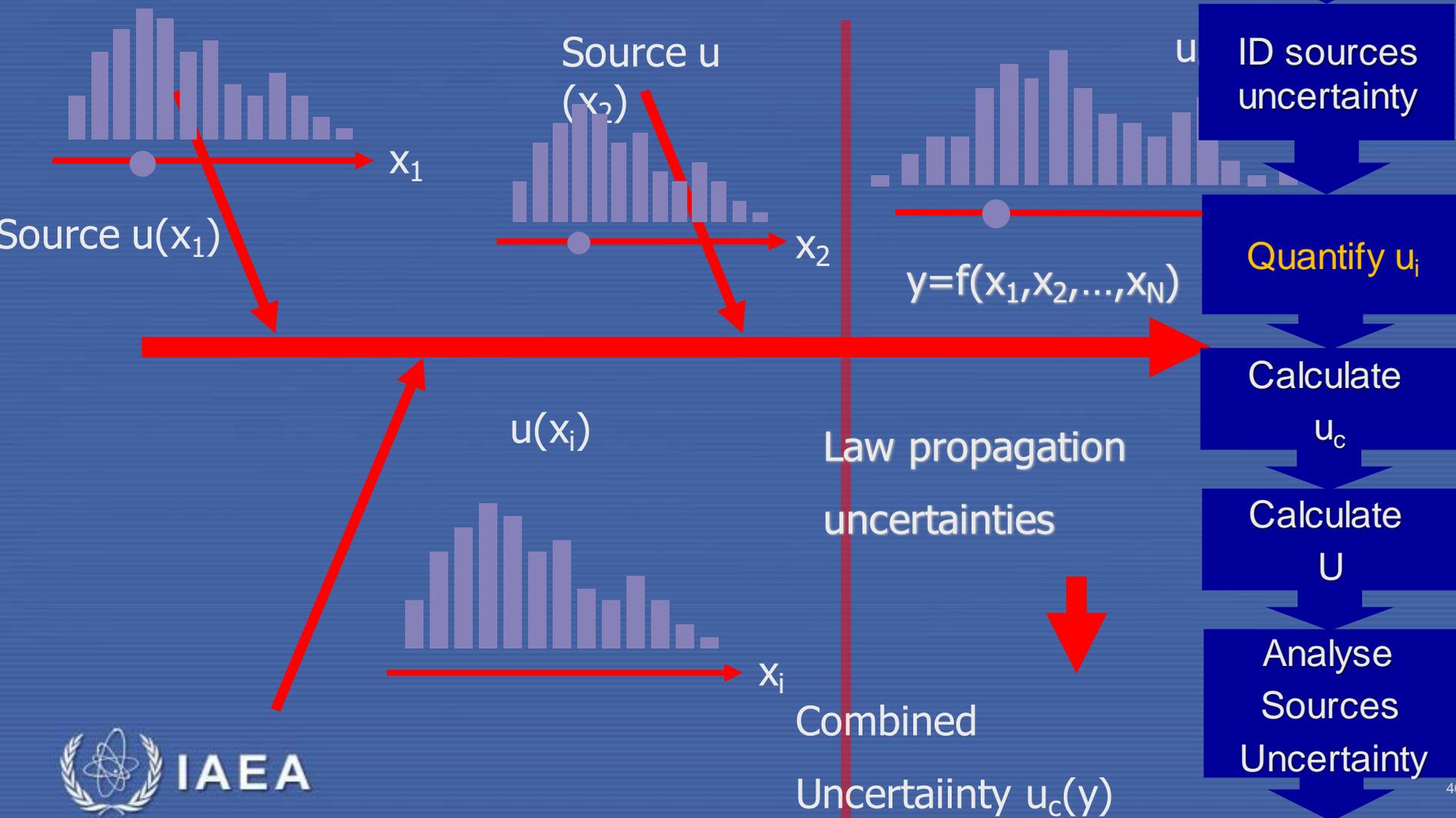
u_c

Calculate

U

Analyse Sources Uncertainty

Uncertainty propagation



Law of the propagation of uncertainties

$$u_c(y) = \sqrt{\sum_{i=1}^n \left[\frac{\partial y}{\partial x_i} \right]^2 u^2(x_i) + 2 \sum_{i=1}^{n-1} \sum_{j=i+1}^n c_i c_j u(x_i) u(x_j) r(x_i, x_j)}$$

$\frac{\partial y}{\partial x_i} = c_i$: sensitivity components

r : correlation coefficients (between 0 and 1)

Define your measurand

ID sources uncertainty

Quantify u_i

Calculate

u_c

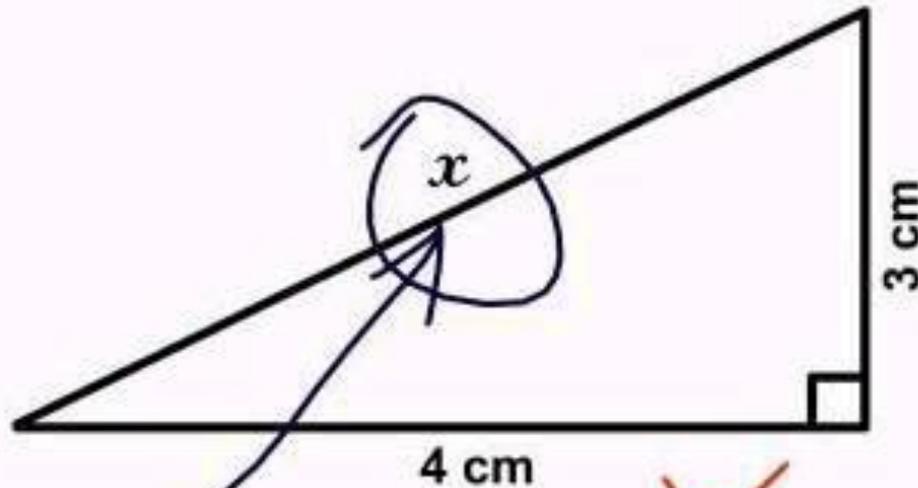
Calculate

U

Analyse Sources Uncertainty

What is RSS = root sum of squares

Find x .



Here it is

~~X~~ O

For a sum or a difference:

$$Y = x_1 + x_2 - x_3 \dots + x_n$$

$$u_c(y) = \sqrt{u_{x1}^2 + u_{x2}^2 \dots + u_{xn}^2}$$

Or the RSS (root sum of squares of the **absolute** standard uncertainties (no RSD's, CV's, ...))

For example, you might need to find the total length of a fence made up of different width fence panels. If the standard uncertainty (in meters) in the length of each fence panel was given by x_1 , x_2 , x_3 , etc., then the combined standard uncertainty (in meters) for the whole fence would be found by the RSS

Define your
measurand

ID sources
uncertainty

Quantify u_i

Calculate

u_c

Calculate

U

Analyse
Sources
Uncertainty

And for multiplication or a division

Is often applicable to most dosimetry systems

$$y = \frac{x_1 \times x_2}{x_3 \times \dots \times x_n}$$

$$u_c(y) = y \sqrt{\left(\frac{u_{x1}}{x_1}\right)^2 + \left(\frac{u_{x2}}{x_2}\right)^2 + \dots + \left(\frac{u_{xn}}{xn}\right)^2}$$

Or therefore RSS of the relative standard uncertainties

For example, you might need to find the area, A, of a rectangular carpet, by multiplying the length, L, by the width, W (i.e. A=L × W). The relative uncertainty in the area of the carpet can be found from RSS of the fractional uncertainties in the length and width (u_L/L and u_W/W).

Define your
measurand

ID sources
uncertainty

Quantify u_i

Calculate

u_c

Calculate

U

Analyse
Sources
Uncertainty

Example of uncertainty budget (Data not real!)

Source of Uncertainty	Type A/B	Value (mSv)	% relative	Probability Distribution	Divisor	v_{eff}	u_i (mSv)	u_{rel}	Comment
Reproducibility RCF	A	2.50	2.4%	Normal	1	4	0.061	2.4%	Validation file
Calibration RCF	A	2.50	6.0%	Normal	2	inf	0.075	3.0%	Cal certificate
Reading repeatability	A	2.50	7.2%	Normal	1	9	0.202	8.1%	Validation file
Reproducibility ECC	A	2.50	1.4%	Normal	1	9	0.034	1.4%	Validation file
Reader blanc indication	A	2.50	1.6%	Normal	1	9	0.039	1.6%	Validation file
Climatic conditions	B	2.50	5%	Rectangular	3.464	inf	0.036	1.4%	Type Testing file Harshaw
Angular dependence	B	2.50	4%	Rectangular	3.464	inf	0.026	1.0%	Validation file

Example of uncertainty budget (Data not real ... Continued)

<i>Energy dependence</i>	<i>B</i>	<i>2.50</i>	<i>69%</i>	<i>Triangular</i>	<i>4.899</i>	<i>inf</i>	<i>0.351</i>	<i>14.0%</i>	<i>Validation file</i>	
<i>Linearity</i>	<i>B</i>	<i>2.50</i>	<i>27%</i>	<i>Triangular</i>	<i>4.899</i>	<i>inf</i>	<i>0.137</i>	<i>5.5%</i>	<i>Validation file</i>	
<i>Light exposure</i>	<i>B</i>	<i>2.50</i>	<i>12%</i>	<i>Rectangular</i>	<i>3.464</i>	<i>inf</i>	<i>0.087</i>	<i>3.5%</i>	<i>Type Testing file Harshaw</i>	
<i>Background Variation</i>	<i>A</i>	<i>1.2895</i>	<i>27%</i>	<i>Normal</i>	<i>1.000</i>	<i>24</i>	<i>0.01050</i>	<i>0.4%</i>	<i>Validation file</i>	
<i>Fading</i>	<i>B</i>	<i>2.50</i>	<i>7%</i>	<i>Rectangular</i>	<i>3.464</i>	<i>inf</i>	<i>0.051</i>	<i>2.0%</i>	<i>Type Testing file Harshaw</i>	
<i>Rounding</i>	<i>B</i>	<i>0.01</i>	<i>100.0%</i>	<i>Rectangular</i>	<i>3.464</i>	<i>inf</i>	<i>0.003</i>	<i>0.1%</i>	<i>Reader</i>	
<i>Combined Standard Uncertainty</i>		<i>2.50</i>					<i>0.45</i>	<i>18%</i>	<i>18%</i>	
<i>Expanded Uncertainty (k=2)</i>		<i>2.50</i>					<i>0.91</i>	<i>36%</i>	<i>36%</i>	

Reporting results

Not Recommended...

- “The measured result is 1,05 mSv \pm 2.8 ppm for a level of confidence of approximately 95%, $k=2$.”

Better...

- “The measured result is 1,05 mSv \pm 0.28 mSv for a level of confidence of approximately 95%, $k=2$.”

- **Better yet...**

- “The measured result is 1,05 mSv \pm 0.28 mSv The reported uncertainty is expanded using a coverage factor $k=2$ for a level of confidence of approximately 95%, assuming a normal distribution.”

Dosimetry Applications

- External dosimetry
(thermoluminescent dosimeters)

- Internal dosimetry
(gamma spectrometric measurements also including whole and partial body counters, liquid scintillation counting, chemical preparation, alpha spectrometry)

Whole Body Counter

Possible contributions to the uncertainty:

- Uncertainty in reference standard
- Uncertainties in construction of a “physical phantom”, homogeneity of the reference material in the phantom
- Detector calibration (efficiency, charge collection, ...)
- Environmental conditions, background radiation
- Individual parameters (size, weight, distribution of the incorporated radionuclides, ...)
- Radionuclide data
- Detector readout, electronics
- Counting statistics

Up to $\pm 25\%$ ^{50%}

Gamma Spectrometry

Possible contributions to the combined uncertainty:

- Uncertainty in reference standard
- Measurement geometry
- Measurement matrix (density, granularity, ...)
- Detector calibration (efficiency, charge collection, ...)
- Environmental conditions, background radiation
- Bias corrections (e.g., cascade summation, etc.)
- Radionuclide data
- Detector readout, electronics
- Counting statistics Down to $\pm 5\%$

Liquid Scintillation Counting

Possible contributions to the combined uncertainty:

- Uncertainty in reference standard
 - Measurement matrix (quench)
 - Detector calibration (efficiency, ...)
 - Detector readout, electronics
 - Radionuclide data
 - Counting statistics
- Down to $\pm 10\%$

Alpha Spectrometry

Possible contributions to the combined uncertainty:

- Uncertainty in reference standard
 - Detector calibration (efficiency, ...)
 - Measurement geometry
 - Environmental conditions
 - Detector readout, electronics
 - Radionuclide data
 - Counting statistics
- Down to $\pm 10\%$