Introduction to Reactivity and Reactor Control

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Learning Objectives

- Define $k$-effective ($k_{\text{eff}}$) and reactivity, and describe their importance in reactor dynamic behavior.
- Describe the units used to represent reactivity.
- Write the time dependent neutron balance and describe what can happen to neutrons in a reactor.
- Describe the important role of delayed neutrons.
- Describe the inherent reactivity effects in a nuclear reactor.
- Describe the natural phenomena and designed systems that can change reactivity.
Learning Objectives

- Describe the signals from process measurements are used to control the reactor.
- Identify and discuss steady state, heat balance relationships in a pressurized water reactor.
- Discuss and critique alternative philosophies for reactor control.
- Understand and illustrate the effects of fission product buildup on reactivity and core kinetics.
- Differentiate among critical, supercritical, and subcritical conditions in a reactor.
Inherent Reactivity Effects

- Reactor Core
  - Kinetics Equations
  - Fuel Burn-up Model
  - Fission Product Poison Model
  - Thermal Hydraulics Model

- Rx Thermal Power
  - \( \rho_{\text{Total}} \)
  - \( \rho_{\text{M}} \)
  - \( \rho_{\text{F}} \)
  - \( \Sigma \)

- S/G
  - \( M_{\text{WE}} \)

- T/G
  - \( T_{\text{M}} \)
  - \( T_{\text{F}} \)

- Symbols:
  - \( \rho_{\text{Total}} \)
  - \( \rho_{\text{BU}} \)
  - \( \rho_{\text{TPoison}} \)
  - \( \rho_{\text{TModerator}} \)
  - \( \rho_{\text{TFuel}} \)
  - \( \alpha_{\text{M}} \)
  - \( \alpha_{\text{F}} \)
External Reactivity Effects by Protection and Control Systems
Signals for Protection and Control

NSSS Control Loops
- Power
  - $T_C$
  - $T_H$
- Control Rods
- Pressure

S/G Control Loops
- Flow
- Level
- Steam Pressure, $P_S$
- RCP
- Feed Flow
- Steam Flow

Turbine – Generator Control Loops
- Voltage
- KVA
- Frequency
- Tachometer
- $1^{st}$ Stage Pressure
- MW$_E$

Core
- Pressure

Pressurizer
In steady state, all heat terms are equal given no losses.
Two balance equations must be satisfied for the reactor to be steady-state:

1. \( \rho_{\text{Total}} = 0 \)

2. \( \dot{Q}_{RX} = \dot{Q}_{S/G} = \dot{Q}_{BoP} \)

The following three equations can be used to calculate each of the above \( Q \) terms:

\[
\begin{align*}
\dot{Q}_{Rx} &= \dot{m}_{RCP} C_p (T_H - T_C) \\
\dot{Q}_{S/G} &= UA(T_{ave} - T_{Steam}) \\
\dot{Q}_{BoP} &= \dot{m}_{FeedWater} (h_{steam} - h_{FeedWater})
\end{align*}
\]
Concepts for Control
One-line Drawing of a PWR

\[ \dot{Q}_{S/G} = UA(T_{Ave} - T_{Steam}) \]

\[ \dot{Q}_{Turbine} = \dot{m}_{Turbine}(h_S - h_X) \]
Response Curves

Constant Tav Program

Advantages:
• Least amount of external control
• Preferred by reactor
• Small pressurizer (minimum expansion of coolant volume as power changes)

Disadvantages
• Drop off of steam temperature and pressure
• Poor turbine efficiency

From Schultz “Control of Nuclear Reactors and Power Plants (1961)
Response Curves

Constant Th Program

Advantages:
• Least stressful to materials

Disadvantages
• Huge drop off of steam temperature and pressure
• Poorest turbine efficiency
• Requires external reactivity control

From Schultz “Control of Nuclear Reactors and Power Plants (1961)

FIG. 8-6. Variations in temperatures and pressure as a function of power output for constant-outlet-temperature program.
Response Curves

Constant Tsteam Program

Advantages:
• Best turbine efficiency
• Preferred by turbine

Disadvantages
• Need large pressurizer volume to accommodate coolant expansion
• Requires external reactivity control

From Schultz “Control of Nuclear Reactors and Power Plants (1961)
Reactor Physics 101
What Can Happen to Neutrons?

1. Fission
   a. Energy Release
      i. prompt
      ii. delayed
   b. Fission Products
   c. More Neutrons
      i. prompt neutrons
      ii. delayed neutrons

2. Capture

3. Scatter

4. Leak
Neutrons emitted during fission can cause additional fission events, creating a self-sustaining chain reaction.
Neutron Balance

\[
\begin{bmatrix}
\text{Rate of Increase in Number of Neutrons}
\end{bmatrix}
= \begin{bmatrix}
\text{Rate of Production of Neutrons}
\end{bmatrix}
- \begin{bmatrix}
\text{Rate of Absorption of Neutrons}
\end{bmatrix}
- \begin{bmatrix}
\text{Rate of Leakage of Neutrons}
\end{bmatrix}
\]

Accumulation = Production − Absorption − Leakage

If Accumulation:
= 0 Critical Steady State Static
> 0 Supercritical Increasing Kinetic/ Dynamic
< 0 Subcritical Decreasing Kinetic/ Dynamic
Effective multiplication and reactivity

$$k_{\text{eff}} = \frac{\text{neutron production rate}}{\text{neutron destruction rate}}$$

Let $P = \text{production rate} = \nu F$ where $F = \text{fission rate}$ and $\nu = \text{number of neutrons per fission}$

Let $A = \text{absorption rate (loss)}$

Let $L = \text{leak rate (loss)}$

Then $k_{\text{eff}} = \frac{P}{A + L}$
Effective multiplication and reactivity

**STATES OF CRITICALITY**

\[ k_{\text{eff}} = 1 \quad \text{Critical} \]
\[ k_{\text{eff}} > 1 \quad \text{Supercritical} \]
\[ k_{\text{eff}} < 1 \quad \text{Subcritical} \]

**DEFINITION OF REACTIVITY, \( \rho \)**

\[
\rho = \frac{k_{\text{eff}} - 1}{k_{\text{eff}}} = \frac{\Delta k}{k} = \frac{P}{A + L} - 1 = \frac{P - A - L}{P} = \frac{\text{net neutron production}}{\text{neutron production}}
\]

\[ \rho = 0 \quad \text{Critical} \]
\[ \rho > 0 \quad \text{Supercritical} \]
\[ \rho < 0 \quad \text{Subcritical} \]
Criticality

- **States of criticality**
  
  \[ k_{\text{eff}} = 1 \quad \text{Critical (}\rho = 0\text{)} \]
  
  \[ k_{\text{eff}} > 1 \quad \text{Supercritical (}\rho > 0\text{)} \]
  
  \[ k_{\text{eff}} < 1 \quad \text{Subcritical (}\rho < 0\text{)} \]

- **No reactor can be constantly critical**
  
  - Fuel depletion
  - Fission product buildup
  - Temperature changes

**Note:** \( k \) and \( k_{\text{eff}} \) are used interchangeably
Critically Control

- States of criticality
  
  \[ k_{\text{eff}} = 1 \quad \text{Critical} \ (\rho=0) \]
  
  \[ k_{\text{eff}} > 1 \quad \text{Supercritical} \ (\rho>0) \]
  
  \[ k_{\text{eff}} < 1 \quad \text{Subcritical} \ (\rho<0) \]

- In order to keep an operating nuclear reactor critical we will need to “adjust” terms in the neutron balance

- Neutron balance controls
  
  - Production
  - Absorption
  - Leakage
Reactivity Units

Units for Reactivity:

- mk (1 mk = 0.001)
- pcm (1 pcm = 0.00001)
- $\Delta \rho$ (given as number without units)
- $\Delta k/k$ (given as number without units)
- $\delta k/k$ (same as $\Delta k/k$)
- $\% \Delta k/k$ ($\Delta k/k \times 100$)
- $\$ = \Delta \rho / \beta$
- $\$ = 0.01$
Energy Released During Fission

FISSION

235\text{U} \rightarrow 143\text{Xe} + 143\text{Cs} + 143\text{Ba} + 143\text{La} + 143\text{Ce} + 143\text{Pr} + 143\text{Nd}

Prompt Energy Release:
- 143\text{Xe}: 0.69s
- 143\text{Cs}: 1.7s
- 143\text{Ba}: 12s
- 143\text{La}: 14 mln
- 143\text{Ce}: 33h
- 143\text{Pr}: 13.6d
- Stable

Delayed Energy Release:
- 90\text{Br}: 1.6s
- 90\text{Kr}: 32.5s
- 90\text{Rb}: 2.7 mln
- 90\text{Sr}: 29y
- 90\gamma: 64h
- 90\text{Zr}: Stable

GAMMA RAYS

FISSION FRAGMENTS

University of Pittsburgh
Nuclear Engineering Program
Neutron Balance with Delayed Neutrons

**Neutron Balance**

The rate of increase in the number of neutrons can be expressed as:

\[
\frac{dn(t)}{dt} = \left(1 - \beta\right)P(t) + \sum_{i=1}^{I} \lambda_i C_i(t) - A(t) - L(t)
\]

**Precursor Balance**

For precursors, the rate of increase is given by:

\[
\frac{dC_i(t)}{dt} = \beta P(t) - \lambda_i C_i(t) \text{ for } i = 1, L, I
\]

Where \(\beta\) is the fraction of neutrons delayed.

\[
\text{Rate of Increase in Number of Neutrons}
\]

\[
\text{Rate of Production of Prompt Neutrons}
\]

\[
\text{Rate of Production of Delayed Neutrons from Precursor Decay}
\]

\[
\text{Rate of Absorption of Neutrons}
\]

\[
\text{Rate of Leakage of Neutrons}
\]

\[
\text{Rate of Increase in Number of Precursors}
\]

\[
\text{Rate of Production of Precursors}
\]

\[
\text{Rate of Radioactive Decay of Precursors}
\]

\[
\text{Rate of Precursor Absorption}
\]

\[
\text{Rate of Precursor Leakage}
\]
Criticality Control (Reactor)

- Nuclear Reactor
  - **Production**
    - Determined by the total fissile content of the core.
    - Initial fuel loading.
  - Absorption
  - Leakage
Criticality Control (Reactor)

- Nuclear Reactor
  - Production
- Absorption
  - Cladding, Structure, Coolant
  - Control Rods
  - Soluble Neutron Absorbers
  - Burnable Neutron Absorbers
  - Fission-Product Absorbtion
- Leakage
Criticality Control (Reactor)

- Nuclear Reactor
  - Production
  - Absorption
    - Modern reactor designs
      - Moveable control rods (CR) to change power level and maintain steady state operation.
      - Movable safety rods (SR) to quickly shut down reactor and ensure $k_{\text{eff}} < 1$.
      - Soluble boron in reactor coolant (PWR only) to “shim” $k_{\text{eff}}$.
      - Fixed burnable absorbers (boron or gadolinium) that deplete during operation.
  - Leakage
Criticality Control (Reactor)

- Nuclear Reactor
  - Production
  - Absorption
  - Leakage
    - Primarily determined by reactor design
    - Modern reactor designs:
      - Use a cylindrical core shape to reduce surface-to-volume ratio while still allowing easy access to fuel
      - Include a material (usually water) surrounding the core to reflect escaping neutron back into the active fuel region of the core
Criticality Control (Reactor)

- **Reactor Criticality Requirements**
  - **Operation Modes**
    - Power Reactors (Startup / Steady-State / Shutdown)
    - All reactors have emergency shutdown (SCRAM or TRIP) capability
  - Routine adjustments to reactor criticality are required
    - Account for power fluctuations and feedback effects
      - Fuel depletion, density changes of moderator
    - Small frequent adjustments: control rods (in PWR)
    - Larger, planned, adjustments: soluble boron (in PWR)
    - BWR reactors use control rods and coolant flow feedback to adjust criticality.
PWR (W & B&W) Control Rod “Spider”
AP600 Core Design

Fuel Assemblies

"Zoom In" on these 4 assemblies

1/4 Core Symmetry
AP600 Assembly Design

Zoom In on Quarter Assembly

Fuel Elements
Control Rods
¼ Assembly
Symmetry
Simplified AP600 Assembly Model

- Simplified 2-D model of an AP600 quarter assembly.
- Contains UO$_2$ fuel, boron control rods, and B4C burnable absorber rods.
- Reflecting boundary conditions on all sides.
Quarter-Assembly, Control Rods Withdrawn

\[ k = 1.1630 \]
Quarter-Assembly, Control Rods Inserted

\[ k = 0.93287 \]
Control Rod Worth Example

INTEGRAL ROD WORTH (pcm)

0 10 20 30

ROD WITHDRAWAL (in.)

DIFFERENTIAL ROD WORTH (pcm/in.)

0 10 20 30

ROD WITHDRAWAL (in.)
Axial Flux w/ Control Rods
ΔI = Upper Power - Lower Power

Want to keep the axial power shape well conditioned

We move control rods to control the axial power shape
  Prevent axial power peaks
Radial Flux w/ Control Rods
PWR Reactivity Control

- Routine Control Rod Adjustment for Critical
  - Full Safety/Control Rod Insertion [Scram/Trip]
    - Overpower
    - Other Parameters Out of Range
- Intermediate / Long-Term
  - Soluble Poison – Boric Acid
    - Minimize Control Rod Use
    - ~25% Group-1 Bite at Full Power
    - Decreased w/ Burnup
    - Changed w/ Steady Power Change
  - Burnable Poison [Shim] Rods
PWR Protective System

- SCRAM / TRIP
  - Full-Length CR Mounted to Drives w/ Electromagnets
  - Loss-of-Current → Full Insertion

- REACTIVITY INVENTORY
  - Control Rods
  - Negative Feedback Defects
  - Shutdown Margin - Several %Δk/k
    - Stuck Rod Criterion - Highest Worth
    - Over Core Lifetime
FEEDBACK EFFECTS

- COEFFICIENTS OF REACTIVITY $\alpha$

$$\alpha(T_i) = \frac{\partial \rho}{\partial T_i}$$

$T_i \rightarrow T_f$ Fuel Temperature Coefficient [FTC]  
$T_m$ Moderator Temperature Coefficient [MTC]  
$f_v$ Moderator Void Coefficient [MVC]  
$d_m$ Moderator Density Coefficient [MDC]

$$feedback \hat{\Delta} \text{reactivity} = \Delta \rho_F = \sum \alpha(T_i) \Delta T_i = \sum \frac{\partial \rho}{\partial T_i} \Delta T_i$$
INTEGRATED SYSTEM RESPONSE

FEEDBACK LOOP

- $\Delta \rho_{\text{EXT}}$ Inserted
- Power $\dot{Q}_{Rx}$ from Kinetics Equations
- $\Delta T$ & $\Delta$ Density $\rightarrow$ Feedback Reactivity $\Delta \rho_{F}$

- $\Delta \rho_{\text{EXT}} + \Delta \rho_{F} = \Delta \rho_{\text{TOTAL}}$

If $\Delta \rho_{\text{TOTAL}} < \Delta \rho_{\text{EXT}}$ $\rightarrow$ Stabilization

If $\Delta \rho_{\text{TOTAL}} > \Delta \rho_{\text{EXT}}$ $\rightarrow$ Unstable $\rightarrow$ Possible System Damage, If Uncompensated
Inherent Reactivity Effects

\[ \Sigma \rightarrow \text{Reactor Core} \rightarrow \text{Kinetics Equations} \rightarrow \text{Rx Thermal Power} \rightarrow \text{S/G} \rightarrow \text{T/G} \rightarrow M_{W_E} \]

- Kinetics Equations
- Fuel Burn-up Model
- Fission Product Poison Model
- Thermal Hydraulics Model

\[ \rho_{\text{Total}} \rightarrow \rho_{\text{BU}} \rightarrow \rho_{T_{\text{Poison}}} \rightarrow \rho_{T_{\text{Moderator}}} \rightarrow \rho_{T_{\text{Fuel}}} \]

\[ \alpha_M \rightarrow T_{\text{Moderator}} \rightarrow T_{\text{Fuel}} \]
Negative Reactivity Feedback Effect

Resists the Effect That Produced It & Is Stabilizing
Positive Reactivity Feedback Effect

Enhances the Effect That Produced It & Is Destabilizing
Fission Product Poisoning - Xenon

We get fission products from the fission process (the ashes of burning uranium)
- Hundreds of various fission products
- All chemical species
- All physical forms
- Some fission products have very large probability to absorb neutrons

When these fission products are present in the reactor, they can have a very strong effect on reactivity, $\rho$
Fission Products (the ashes) Released During Fission

**FISSION**

- $^{235}\text{U}$
- $\text{FISSION FRAGMENTS}$
- $\gamma$
- $\beta^-$
- $\gamma$
- $\beta^-$
- $\gamma$
- $\beta^-$
- $\gamma$
- $\beta^-$
- $\gamma$
- $\beta^-$
- $\gamma$
- $\beta^-$
- $\gamma$
- $\beta^-$

- $^{90}\text{Br}$: $1.6\text{s}$
- $^{90}\text{Kr}$: $32.5\text{s}$
- $^{90}\text{Rb}$: $2.7\text{min}$
- $^{90}\text{Sr}$: $29\text{y}$
- $^{90}\gamma$: $64\text{h}$
- $^{90}\text{Zr}$: stable

- $^{143}\text{Xe}$: $0.69\text{s}$
- $^{143}\text{Cs}$: $1.7\text{s}$
- $^{143}\text{Ba}$: $12\text{s}$
- $^{143}\text{La}$: $14\text{mln}$
- $^{143}\text{Ce}$: $33\text{h}$
- $^{143}\text{Pr}$: $13.6\text{d}$
- $^{143}\text{Nd}$: stable

Prompt Energy Release

Delayed Energy Release
Fission Product Poisoning - Xenon

- When a new reactor with fresh fuel starts up, poisoning by xenon is not evident until some Xe has formed.

- Xe builds up to an equilibrium level and absorbs neutrons.

- When the reactor is shut down, it goes through a peaking transient that can affect the ability to restart the reactor.
Fission Product Poisoning - Xenon

- Production of Xenon-135
  - Produced in two ways:
    - As a daughter in a radioactive decay chain (from Iodine-135,
    - As a direct yield from fission
  - Lost in two ways:
    - By radioactive decay,
    - By absorption of a neutron to become Xe-136 (weak absorber)
$^{135}$Xe Production

$^{135}$Te $\rightarrow^{\beta^-}$ 19 s $^{135}$I

$^{135}$I $\rightarrow^{\beta^-}$ 6.6 h $^{135}$Xe

$^{135}$Xe $\rightarrow^{2.6 \times 10^6}$ b $^{136}$Xe

$^{135}$Xe $\rightarrow^{9.1}$ h $^{135}$Cs

$^{135}$Cs $\rightarrow^{2.3 \times 10^6}$ $^{135}$Ba

Table:

<table>
<thead>
<tr>
<th>FISSION NUCLIDE</th>
<th>$\gamma^{(135\text{Te})}$</th>
<th>$\gamma^{(135\text{Xe})}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{233}$U</td>
<td>0.051</td>
<td></td>
</tr>
<tr>
<td>$^{235}$U</td>
<td>0.061</td>
<td>0.003</td>
</tr>
<tr>
<td>$^{239}$Pu</td>
<td>0.055</td>
<td></td>
</tr>
</tbody>
</table>
FISSION PRODUCTS

XENON-135 Production (P) and Destruction

\[
\begin{align*}
\left\{ \text{Rate of change of Xenon} \right\} &= \left\{ P_{\text{Xenon}} + \text{Decay}_{\text{Iodine}} \right\} - \left\{ \text{Burnup}_{\text{Xenon}} + \text{Decay}_{\text{Xenon}} \right\} \\
\left\{ \text{Rate of change of Iodine} \right\} &= \left\{ P_{\text{Iodine}} \right\} - \left\{ \text{Decay}_{\text{Iodine}} \right\}
\end{align*}
\]
The Transient Xenon Problem

Here it is in a picture:

But there are other things going on:

1. Production processes from fission which we represent as a valve whose opening is proportional to the flux (power) level.
2. Burnup loss goes to zero if power (flux) goes to zero.
Fission Product Poisoning - Xenon

Major Points to note:

1. Xenon effects will be felt over relatively long time intervals (since Xe and I decay so slowly).

2. Production of Xenon is from Iodine decay and direct fission yield.

3. Iodine decays faster than Xenon
Effect of Xenon on Reactivity

What does this mean when we operate a reactor?

1. We start up a new reactor. It is xenon-free.
2. We go critical at low power (low flux) - negligible xenon
3. We bring the reactor to high power
4. As xenon builds up, we have to withdraw control rods to stay critical

Xenon reactivity worth could be on the order of

\[ \rho_{Xe} \approx -0.024 = -24 mk \]
$^{135}$Xe Behavior

$^{135}$Xe CONCENTRATION at/cm$^3$

INITIAL STARTUP

TIME FROM INITIAL STARTUP, h

$3 \times 10^{15}$

$2 \times 10^{15}$

$1 \times 10^{15}$

$0$
$^{135}\text{Xe}$ Behavior

![Graph showing $^{135}\text{Xe}$ concentration over time.

Axes:
- Y-axis: $^{135}\text{Xe}$ concentration in atoms/cm$^3$.
- X-axis: Time from initial startup in hours.

Key points:
- Initial Startup
- Shutdown

Concentration values:
- $3 \times 10^{15}$ atoms/cm$^3$
- $2 \times 10^{15}$ atoms/cm$^3$
- $1 \times 10^{15}$ atoms/cm$^3$
- $0$ atoms/cm$^3$
OPERATIONAL IMPACTS

LONG-TERM REACTIVITY CONTROL

Programmed Control Rod Motion

- Change Power Level
  - Startup / Shutdown
  - Load Follow
  - Re-Start

- Withdraw to Compensate Fuel Burnup
- Damp Xenon Oscillations
- Concern → Power Peaking
OPERATIONAL IMPACTS

LONG-TERM REACTIVITY CONTROL

Soluble Poisons
- Inject / Dilute to Match Power Level
- Dilute to Compensate Fuel Burnup
- Reduce Control Rod Use
- Concern → Positive Coolant/Moderator Feedback
Inherent Reactivity Effects

Reactor Core

Kinetics Equations

Rx Thermal Power

S/G

T/G

MWE

走势

Fuel Burn-up Model

Fission Product Poison Model

Thermal Hydraulics Model

\(\sum\)

\(\rho_{\text{Total}}\)

\(\rho_{\text{BU}}\)

\(\rho_{T_{\text{Poison}}}\)

\(\rho_{T_{\text{Moderator}}}\)

\(\rho_{T_{\text{Fuel}}}\)

\(\alpha_M\)

\(\alpha_F\)

\(T_{\text{Moderator}}\)

\(T_{\text{Fuel}}\)
External Reactivity Effects by Protection and Control Systems

Rx Protection System

ρ_{Trip}  ρ_{Rods}  ρ_{Boron}  ρ_{Total}  ρ_{Inherent}

NSSS Rx Core

Kinetics Equations

Control System

Control Functions

S/G  T/G  MW_{E}

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