A NEW METHOD TO DETECT CRACK INITIATION FROM STUDIES OF CRACK-PROPAGATION $\Delta a/\Delta N$

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The values of crack propagation (Fig. 7) da/dN are really a quotient of the difference $\Delta a/\Delta N$. In order to get the differential quotient (infinite element with arbitrary unit)

$$\lim_{\Delta N \to o} \frac{\Delta a}{\Delta N} \longrightarrow \frac{da}{dN} = m = a' \qquad (1)$$

at any point "Δa" it is necessary to have a function $\Delta a = f(N)$ but this function is unknown. With the following method it is proposed how we can obtain the number N_i of cycles for crack initiation.

One obtains graphically the curve of $\Delta a = f(N)$ when the total number of cycles are summed up (integrated). It is evident that the correct trend is obtained for $\Delta a/\Delta N$ decreasing with decreasing Δa or with increasing cycles N for a given arbitrary unit of Δa or ΔN respectively. To define da/dN at any point Q on the curve, the tangent with slope m is necessary. However, the curve is not fully mathematically defined and also one cannot draw the tangent graphically; this is only possible at the point of origin of the ordinate $\Delta a = 0$. Here the slope m is exactly defined namely $da/dN = m = 0$. Next the abscissa is taken for numbers of cycles and then $\Sigma N = N_{total} \hat{=} 0'$ is attached at point $0' \hat{=} E^*$ as shown in Fig. 1. This corresponds to the total $\Delta a = 15$ mm in this case. Now descending from 0' to $Q_0 \hat{=} N_i$ in the contrary direction one cuts the axis of numbers of cycles orthogonally with the slope $m = da/dN = 0$ and thus at point Q_0 in Fig. 1 we get the number of cycles for initiation N_i^c.

In calculations, this operation is called an inversion (in German "Spiegelung") and is found by substituting for example a given quotient q by $-q^{-1}$.

Assuming a function of $a = a(N)$, $\underline{da/dN = a' = a(N) \sim a/N}$ one obtains the orthogonal trajectories in (1) by substituting

$$a' \text{ by } -1/a' \quad \text{or} \quad a/N \text{ by } -N/a \qquad (1a)$$

In Fig. 1 there exist in each point Q the accompanying bihedral formed by the vector of unit $\vec{t}$ and the normal vector of unit $\vec{n}$ ($\vec{n} \perp \vec{t}$), turning with a moving Q on the curve. $\vec{t}$ has the direction of the slope m.

The normal vector $\vec{n}$ belongs to the curve, which cuts orthogonally the given function $a = f(N)$ and $\vec{n}$ is at the same time the vector of tangent for the inverted curve $N = f(a)$ with a negative slope m* ($m^* \perp m$).

The relating fixed system of coordinates is defined as origin with the point $Q_0 \triangleq N_i$ with slope $m \equiv 0$.

Instead to go the indirect way from one point to another you can go directly the way E^*Q_0 by a geodetic line.

PRACTICAL CONSEQUENCE WITH THE NEW METHOD

In case of a load controlled experiment with a given load P_i we have measured crack initiation optically at 0.2 mm without extrapolation and have counted 23 cycles. We have then proceeded to $\Sigma\, 37 \triangleq 0'$ cycles and after breaking the specimen, we have measured $\Delta a \sim 2.3$ mm, that means $\Delta a = 2.3$ mm $\triangleq \Sigma 37$ cycles N. Using the proposed method as shwon in Fig. 1 we get initiation N_i^c after 12 cycles.

When this method is acceptable we can do mechanical tests without further equipment and we can define initiation from a given crack growth at any temperature or in chemical solutions where the application of foils is not possible.

With this newly defined crack initiation (which probably starts as a nucleus in the plastic zone) it is possible to plot the valid quantity K^i_{Ic} as a function of N_i as shown in Fig. 2. In this case we take $a = 20$ mm according to machined length in Fig. 4 making an error $\ll 1$ % for calculating

$$K = \frac{P_i}{BW^{1/2}} \; f\,(a/w) \qquad (2)$$

according to ASTM E 399. Tensile tests at $N = 1/2$* with only tension can be also included, Fig. 3a/b. In this case using a compliance method as a partial

*In case of tension and compression N = 1/4.

elastic unloading method one doesn't detect any deviation before the maximum load; thus we have taken P_{max} for calculation of K_q. From work stemming from studies in the field of J-integral-measurement we have to assume that crack initiation lies either before the maximum load point P_{max} or at P_{max}. This throws still some doubt on existing K-values calculated with P_{max} as load. Another problem existing at the moment in fracture mechanics is the dependence of K on the thickness B: One can see at the ordinate from Fig. 2 that K decreases with increasing B-values.

CONCLUSION

With the Wöhler diagram $K = f(N_i)$ one has the total spectrum of specific tests for K_o from tensile test (fracture mechanics) over low cycle fatigue ($10^3 - 10^4$ cycles) to high cycle fatigue until $10^6 - 10^8$ cycles with simple test methods.

At the moment the problems are:

- Crack initiation in tensile tests in function of thickness B for a given structure of material
- The influence of frequency
- The behaviour of different radii with constant a/w with the aim to correlate with smooth specimens in order to convert stress intensity factor $\Delta K \rightarrow$ stress σ.

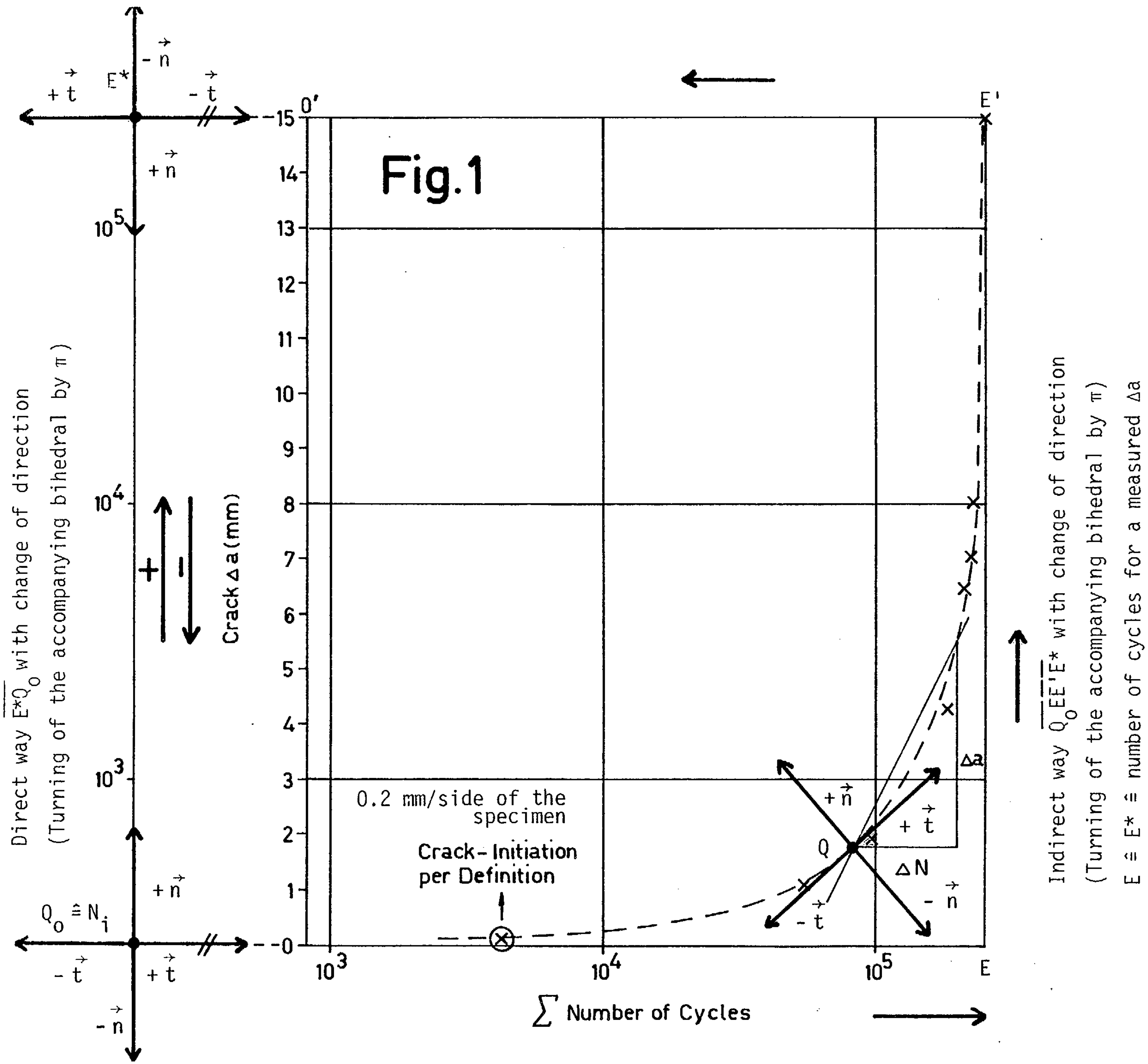


Fig.1
Crack Δa (mm)
Σ Number of Cycles
0.2 mm/side of the specimen
Crack-Initiation per Definition
Q
ΔN
Δa
0'
E'
E
E*
$Q_o \hat{=} N_i$
+t
-t
+n
-n
Direct way $\overline{E^*Q_o}$ with change of direction
(Turning of the accompanying bihedral by π)
Indirect way $\overline{Q_oEE'E^*}$ with change of direction
(Turning of the accompanying bihedral by π)
E ≙ E* ≡ number of cycles for a measured Δa

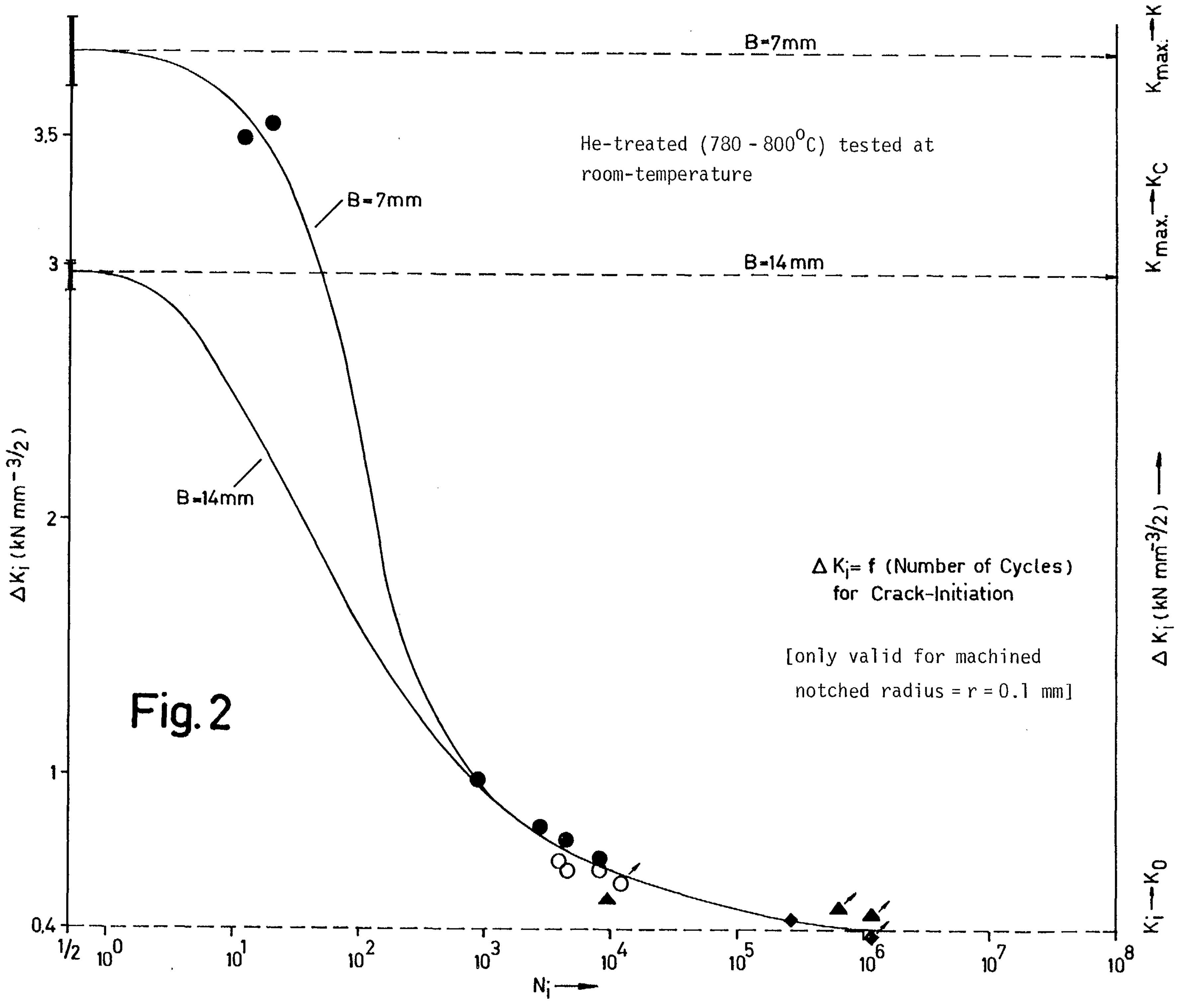

B=7mm
B=14mm
He-treated (780 - 800°C) tested at
room-temperature
B=7mm
B=14mm
Δ Ki= f (Number of Cycles)
for Crack-Initiation
[only valid for machined
notched radius = r = 0.1 mm]
Fig. 2
ΔKi (kN mm-3/2)
3,5
3
2
1
0.4
1/2
10^0
10^1
10^2
10^3
10^4
10^5
10^6
10^7
10^8
Ni
Kmax.→K
Kmax.→Kc
ΔKi (kN mm-3/2)
Ki→K0

Fig. 3b

Tensile test at precracked specimens
700-800 °C/1000^h

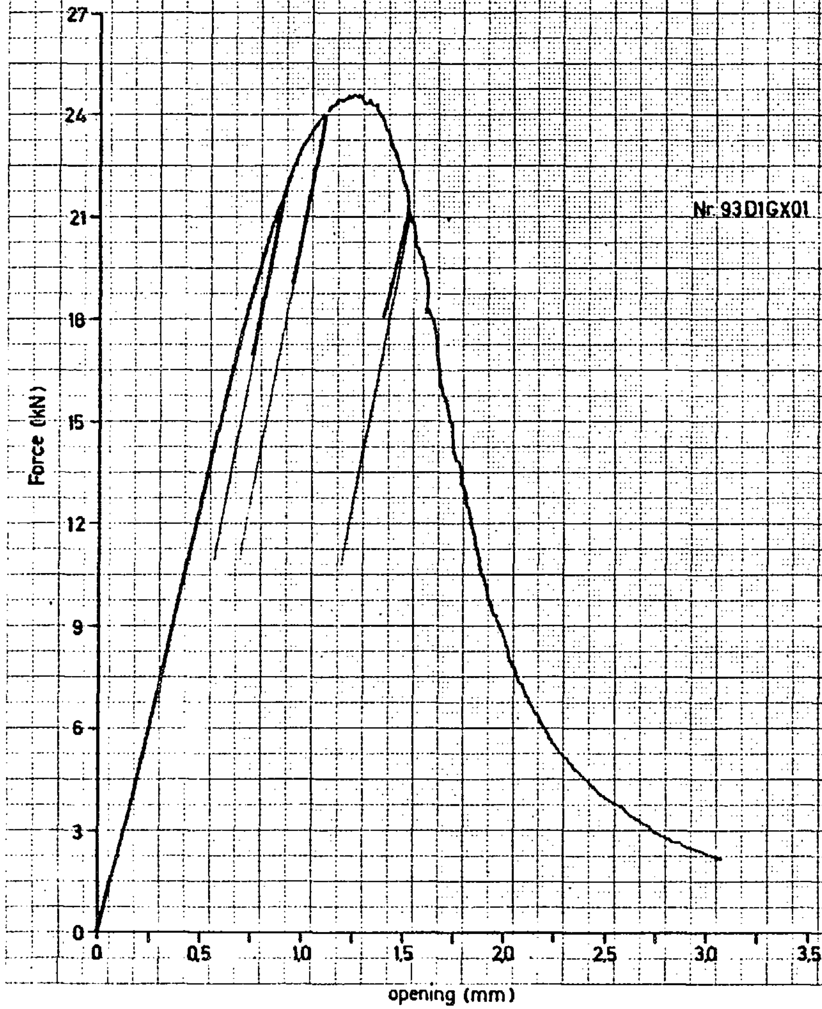
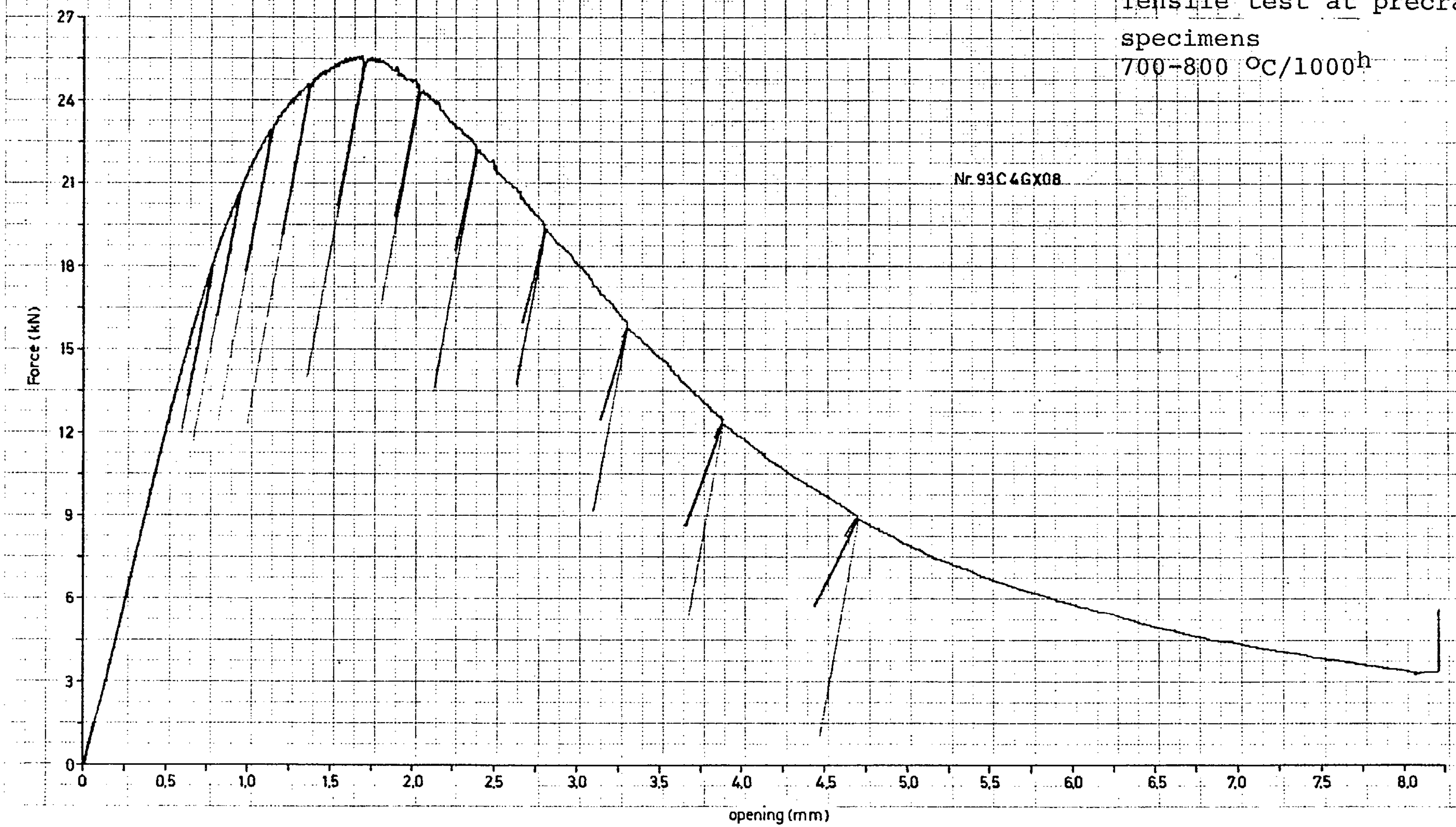


Fig. 3 a

Tensile test at precracked specimens
700-800 °C/1000^{h}



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