Fluid-fuel systems

Summary of lecture

► General model

► Discussion of dynamic effects of fuel motion

► Derivation of consistent point kinetics

► Quasi-static scheme

The neutron kinetics of circulating-fuel reactors

Balance equations for neutrons:

$$\begin{cases} \dfrac{\partial n(\mathbf{r},E,\Omega,t)}{\partial t} = \left[\hat{\mathrm{L}}(t)+\hat{\mathrm{M}}_p(t)\right] n(\mathbf{r},E,\Omega,t) + \\ \qquad \displaystyle\sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r},E,t) + S(\mathbf{r},E,\Omega,t) \\ \\ \dfrac{1}{\lambda_i}\dfrac{\partial \mathcal{E}_i(\mathbf{r},E,t)}{\partial t} + \dfrac{1}{\lambda_i}\boldsymbol{\nabla}\cdot\left(\mathbf{u}\mathcal{E}_i(\mathbf{r},E,t)\right) = \\ \qquad \hat{\mathrm{M}}_i(t) n(\mathbf{r},E,\Omega,t) - \mathcal{E}_i(\mathbf{r},E,t), \\ \\ \qquad\qquad i = 1, 2, ..., R, \end{cases}$$

$$\mathcal{E}_i(\mathbf{r},E,t) = \lambda_i C_i(\mathbf{r},t)\frac{\chi_i(E)}{4\pi}.$$

In the equations for precursors a convective term appears due to fuel motion, and appropriate boundary conditions must then be introduced:

$$\mathcal{E}_i(\mathbf{r}, E, t)\mathbf{u}\,(\mathbf{r}) \cdot (-\mathbf{n}) =$$

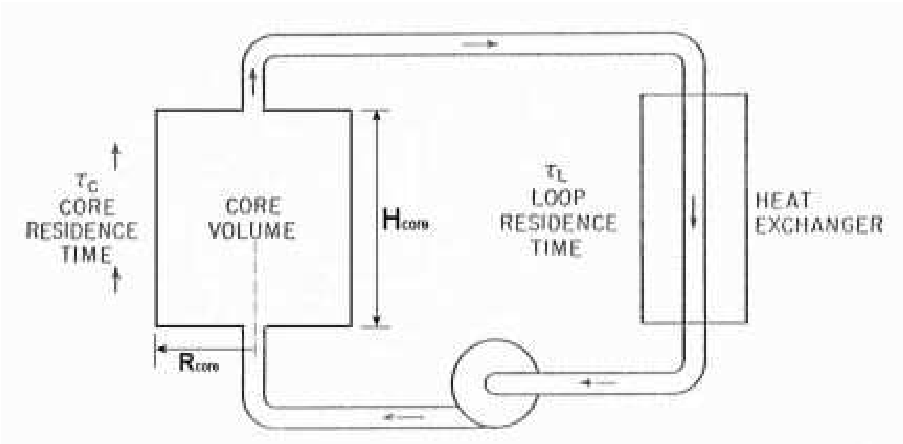
$$\int_{\mathcal{A}_{out}} \mathcal{E}_i(\mathbf{r}', E, t - \tau(\mathbf{r}' \to \mathbf{r}))e^{-\lambda_i \tau(\mathbf{r}' \to \mathbf{r})} \times$$

$$\mathbf{u}\left(\mathbf{r}'\right) \cdot \mathbf{n}' \mathfrak{F}\left(\mathbf{r}' \to \mathbf{r}\right) d\mathcal{A}',$$

$$\mathbf{r} \in \mathcal{A}_{in}.$$

Fission products are moved through and outside the core by the motion of the fissile material.

Geometrical structure of a circulating-fuel reactor



Multigroup diffusion model in cilindrical gometry + slug flow

In slug-flow conditions the velocity field is maintained by externally-driven devices and is one-dimensional (axial):

$$\nabla \cdot (C_i \mathbf{u}) = \frac{\partial}{\partial z}(uC_i)$$

$$\tau_c = \frac{H_{core}}{\bar{u}_{core}} \qquad \tau_L = \frac{L_{loop}}{\bar{u}_{loop}}$$

$$\frac{\bar{u}_{core}}{\bar{u}_{loop}} = \frac{\mathcal{A}_{loop}}{\mathcal{A}_{core}} \qquad \mathcal{A}_{core} = \pi R_{core}^2$$

$$\frac{1}{v_g}\frac{\partial \Phi_g}{\partial t} = \nabla \cdot D_g \nabla \Phi_g - \Sigma_g \Phi_g +$$
$$\sum_{g'} \left[\chi_{p,g} \nu \Sigma_{fg'} (1-\beta) + \Sigma_{g' \to g}\right] \Phi_{g'} +$$
$$S_g + \sum_i \lambda_i \chi_{i,g} C_i$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \sum_g \nu \Sigma_{fg} \Phi_g - \frac{\partial}{\partial z}(u C_i)$$

⋆ boundary conditions:

$$u(0) C_i(z = 0, r, t) =$$
$$u(H) \frac{e^{-\lambda_i \tau_L}}{\mathcal{A}_{core}} \int_{\mathcal{A}_{core}} C_i(z = H, r, t - \tau_L) d\mathcal{A}$$

Discussion of dynamic effects of motion

i) the delayed-precursor equations cannot be eliminated in steady-state configuration.

solid fuel: the concentrations of precursors can be expressed as functions of the fission term and substituted into the neutron balance equation

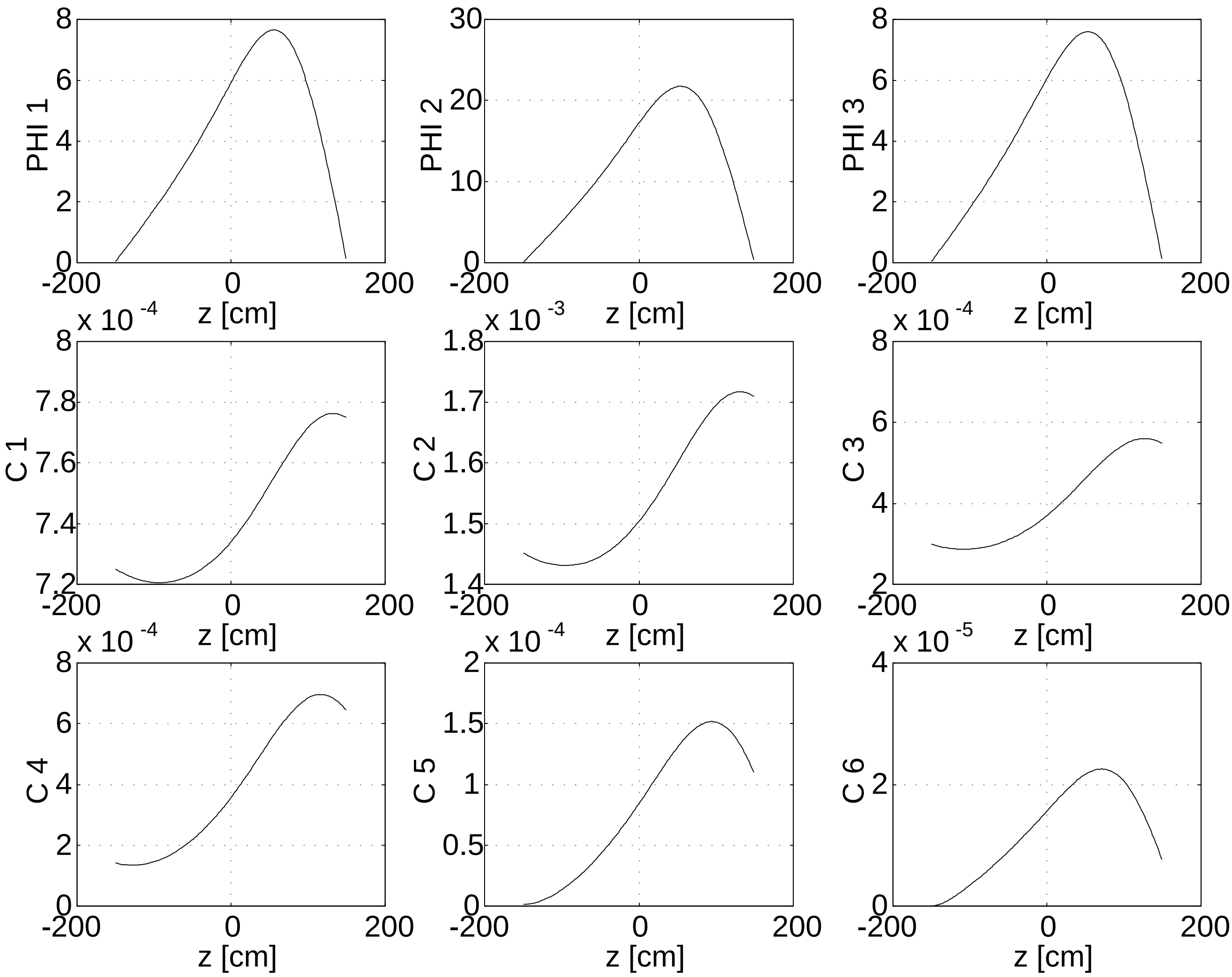
$$\mathcal{E}_i(\mathbf{r}, E) = \hat{\mathrm{M}}_i n(\mathbf{r}, E, \Omega)$$

circulating fuel: the equations for precursors are still differential for the space variable and their concentrations can not be made explicit and substituted

$$\boldsymbol{\nabla} \cdot (\mathbf{u}\mathcal{E}_i(\mathbf{r}, E)) = \lambda_i \hat{\mathrm{M}}_i n(\mathbf{r}, E, \Omega) - \lambda_i \mathcal{E}_i(\mathbf{r}, E)$$

ii) the multiplication eigenvalue depends on delayed neutron and flow characteristics;

iii) the space distribution of delayed-precursors is not following the neutron distribution and is completely different from standard solid-fuel systems


PHI 1
PHI 2
PHI 3
C 1
C 2
C 3
C 4
C 5
C 6
z [cm]
x 10 -4
x 10 -3
x 10 -5

iv) the role of delayed emissions is reduced by space-redistribution and external recirculation (reduction of effective β)

$$\beta_{i,eff} = \frac{\lambda_i \left\langle \Phi^+ \right| C_i \rangle}{\sum_j \lambda_j \left\langle \Phi^+ \right| C_j \rangle + (1-\beta) \left\langle \Phi^+ \right| \nu \Sigma_f \Phi \rangle}$$

Ratio $\tilde{\beta}/\beta$ as a function of τ_L and k_{eff}

τ_L [s] $\longrightarrow$	0	5	10	15	u [cm/s] $\downarrow$
$k_{eff} = 0.95009$	0.842	0.542	0.470	0.443	60
	0.835	0.422	0.363	0.332	100
$k_{eff} = 0.97048$	0.843	0.540	0.469	0.441	60
	0.834	0.420	0.353	0.330	100
$k_{eff} = 0.99028$	0.842	0.539	0.467	0.440	60
	0.833	0.419	0.352	0.329	100
$k_{eff} = 1.00001$	0.843	0.539	0.468	0.440	60
	0.834	0.420	0.352	0.329	100

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v) factorization schemes should be applied to both neutrons and precursors

vi) point kinetics model needs a specific formulation

vii) the numerical solution can take advantage of the slower time-scale of the motion of the delayed precursors

Point kinetic model for circulating fuel systems (consistent with Henry factorization procedure**)**

$\Rightarrow$ the time-dependent balance equations for neutrons and delayed precursors are considered:

$$\begin{cases} \dfrac{\partial n(\mathbf{r}, E, \Omega, t)}{\partial t} = \left[\hat{\mathrm{L}}(t) + \hat{\mathrm{M}}_p(t)\right] n(\mathbf{r}, E, \Omega, t) + \\ \qquad \displaystyle\sum_{i=1}^{R} \mathcal{E}_i(\mathbf{r}, E, t) + S(\mathbf{r}, E, \Omega, t) \\ \\ \dfrac{1}{\lambda_i} \dfrac{\partial \mathcal{E}_i(\mathbf{r}, E, t)}{\partial t} + \dfrac{1}{\lambda_i} \boldsymbol{\nabla} \cdot (\mathbf{u} \mathcal{E}_i(\mathbf{r}, E, t)) = \\ \qquad \hat{\mathrm{M}}_i(t) n(\mathbf{r}, E, \Omega, t) - \mathcal{E}_i(\mathbf{r}, E, t), \\ \qquad\qquad i = 1, 2, ..., R, \end{cases}$$

+initial and boundary condition

$\Rightarrow$ a reference configuration is introduced

$$
\begin{cases}
0 = \left[\hat{\mathrm{L}}_0 + \hat{\mathrm{M}}_{p,0}\right] N_0(\mathbf{r}, E, \Omega) + \\
\qquad \sum_{i=1}^{R} \mathcal{E}_{i,0}(\mathbf{r}, E) + S_0(\mathbf{r}, E, \Omega) \\
\\
\frac{1}{\lambda_i} \boldsymbol{\nabla} \cdot \left(\mathbf{u}_0 \mathcal{E}_{i,0}(\mathbf{r}, E)\right) = \\
\qquad \hat{\mathrm{M}}_{i,0} N_0(\mathbf{r}, E, \Omega) - \mathcal{E}_{i,0}(\mathbf{r}, E), \\
\qquad\qquad i = 1, 2, ..., R,
\end{cases}
$$

+boundary conditions

⇒ a physically consistent definition of the neutron concentrations and delayed emissions importance $n^\dagger$ and $\mathcal{E}^\dagger$ is introduced;

⇒ the balance equations for the importance functions are shown to be the mathematical adjoint to the balance equations for neutrons and precursors, having defined the inner product as:

$$
\begin{aligned}
(\mathbf{w}^\dagger, \mathbf{w}) = & \sum_{n=1}^{R+1} \langle w_n^\dagger \mid w_n \rangle = \\
& \sum_{n=1}^{R+1} \int_V dV \int_E dE \int_{4\pi} d\Omega \cdot w_n^\dagger w_n
\end{aligned}
$$

where

$$
\begin{aligned}
\mathbf{w} &= (n, \mathcal{E}_1, ..., \mathcal{E}_R)^t \\
\mathbf{w}^\dagger &= (n^\dagger, \mathcal{E}_1^\dagger, ..., \mathcal{E}_R^\dagger)
\end{aligned}
$$

$\Rightarrow$ The system of equations for the importance takes the form:

$$\begin{cases} \left[\hat{\mathrm{L}}_0^+ + \hat{\mathrm{M}}_{p,0}^+\right] N_0^+(\mathbf{r}, E, \Omega) + \sum_{i=1}^{R} \hat{\mathrm{M}}_{i,0}^+ \mathcal{E}_{i,0}^+(\mathbf{r}, E) + \\ \qquad S_0^+(\mathbf{r}, E, \Omega) = 0, \\ \\ N_0^+(\mathbf{r}, E, \Omega) + \frac{1}{\lambda_i} \mathbf{u}_0 \cdot \boldsymbol{\nabla} \left(\mathcal{E}_{i,0}^+(\mathbf{r}, E)\right) - \\ \qquad \mathcal{E}_{i,0}^+(\mathbf{r}, E) = 0, \qquad i = 1, 2, ..., R, \end{cases}$$

with boundary conditions:

$$\mathcal{E}_i^+(\mathbf{r}, E) = \int_{\mathcal{A}_{in}} \mathcal{E}_i^+(\mathbf{r}', E) e^{-\lambda_i \tau(\mathbf{r} \to \mathbf{r}')} \mathfrak{F}\left(\mathbf{r} \to \mathbf{r}'\right) d\mathcal{A}',$$

$$\mathbf{r} \in \mathcal{A}_{out}.$$

$\Rightarrow$ Both flux and delayed emission distributions are factorized with an amplitude-shape formula:

$$n(\mathbf{r}, E, \Omega, t) = A(t)\varphi(\mathbf{r}, E, \Omega; t),$$
$$\mathcal{E}_i(\mathbf{r}, E, t) = G_i(t)e_i(\mathbf{r}, E; t) \qquad i = 1, 2, ..., R.$$

* The factorized formula is introduced into balance equations;

* A projection on the adjoint solution is taken;

* A normalization conditions is imposed on the shape functions to make the factorization unique:

$$\frac{d}{dt}\left\langle N_0^+ \middle| \varphi \right\rangle = 0,$$

$$\frac{d}{dt}\left\langle \mathcal{E}_{i,0}^+ \middle| e_i \right\rangle = 0, \qquad i = 1, 2, ..., R,$$

$\Longrightarrow$ a point-like model is obtained

$$\begin{cases} \Lambda_P \dfrac{dA}{dt} = \left(\rho_S - \tilde{\beta}\right) A + \displaystyle\sum_{i=1}^{R} \lambda_i \Gamma_i + \tilde{S} \\[2em] \Lambda_i \dfrac{d\Gamma_i}{dt} = \left(\tilde{\beta}_i + \rho_i\right) A - (\lambda_i + \mu_{u,i} + \mu_{\xi,i}) \Gamma_i + \sigma_i \\[2em] i = 1, ..., R \end{cases}$$

$\rightarrow$ structure equivalent to the point kinetic model for solid-fuel systems;

$\rightarrow$ kinetic parameters and effective delayed neutron functions Γ_i have different definitions;

$\rightarrow$ unconventional terms:

ρ_i : perturbation of production;

$\mu_{u,i}$: perturbation of fluid velocity;

$\mu_{\xi,i}$: perturbation of recirculation time;

- normalization factor

$$\mathcal{F} = \sum_{i=1}^{R} \sum_{n=1}^{G} \left\langle \Phi_n^{\dagger} \right| \chi_{i,n} \lambda_i C_{i,0} \rangle +$$

$$+ (1-\beta) \sum_{n=1}^{G} \sum_{g=1}^{G} \left\langle \Phi_n^{\dagger} \right| \chi_n \left(\nu \Sigma_f \right)_{g,0} \Phi_{g,0} \rangle$$

- effective delayed neutron precursors

$$\Gamma_i = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^{\dagger} \right| \chi_{i,n} C_{i,\,0} \rangle \, G_i(t)$$

- effective neutron source

$$\tilde{S} = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^{\dagger} \right| S_n \rangle$$

- effective delayed neutron fractions

$$\tilde{\beta}_i = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^{\dagger} \right| \chi_{i,n} \lambda_i C_{i,0} \rangle \, , \qquad \tilde{\beta} = \sum_{i=1}^{R} \tilde{\beta}_i .$$

- reactivity $\rho_S = \rho_0 + \rho_{pert}$, where

$$\rho_0 = -\frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle S_n^\dagger \right| \Phi_{n,0} \rangle$$

$$\rho_{pert} = \frac{1}{\mathcal{F}} \left\{ -\sum_{n=1}^{G} \left\langle \boldsymbol{\nabla} \Phi_n^\dagger \cdot \right| \delta D_n \boldsymbol{\nabla} \Phi_{n,0} \rangle + \right.$$

$$+ \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \sum_{g=1}^{G} (1-\beta) \chi_n \delta \left(\nu \Sigma_f \right)_g \Phi_{g,0} \Big\rangle +$$

$$- \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \delta \Sigma_{R,n} \Phi_{n,0} \rangle +$$

$$\left. + \sum_{n=1}^{G} \sum_{g=1}^{G} \left\langle \Phi_n^\dagger \right| \delta \Sigma_{g \to n} \Phi_{g,0} \rangle \right\}$$

- prompt neutron lifetime

$$\Lambda_P = \frac{1}{\mathcal{F}} \sum_{n=1}^{G} \left\langle \Phi_n^\dagger \right| \frac{1}{v_n} \Phi_{n,0} \Big\rangle$$

- generalized precursor lifetime

$$\Lambda_i = \frac{\left\langle C_i^\dagger \middle| C_{i,0} \right\rangle}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} C_{i,0} \right\rangle}; \quad i = 1, 2, ..., R$$

- unconventional terms

$$\rho_i = \frac{1}{\mathcal{F}} \left\langle C_i^\dagger \middle| \beta_i \sum_{g=1}^{G} \delta(\nu\Sigma_f)_g \Phi_{g,0} \right\rangle$$

$$\mu_{u,i} = \frac{\left\langle C_i^\dagger \middle| \frac{\partial}{\partial z} \left(\delta u C_{i,0}\right) \right\rangle}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} C_{i,0} \right\rangle}$$

$$\mu_{\xi,i} = \frac{u_0(H) \int_{\mathcal{A}_{core}} C_i^\dagger(H) C_{i,0}(r,H) d\mathcal{A}}{\sum_{n=1}^{G} \left\langle \Phi_n^\dagger \middle| \chi_{i,n} C_{i,\,0} \right\rangle}$$

- apparent precursor source

$$\Xi = \frac{1}{\mathcal{A}_{core}} u_0(H) \left(\xi_{i,0} + \delta\xi_i\right) \times \int\limits_{\mathcal{A}_{core}} C_i^{\dagger}(r,0) d\mathcal{A} \int\limits_{\mathcal{A}_{core}} C_{i,\,0}(r,H)$$

$$\sigma_i = \begin{cases} \dfrac{\Xi}{\sum\limits_{n=1}^{G} \left\langle \Phi_n^{\dagger} \right| \chi_{i,n} C_{i,0} \rangle} \Gamma_{i,0}, & \text{if } t \le T_R, \\ \dfrac{\Xi}{\sum\limits_{n=1}^{G} \left\langle \Phi_n^{\dagger} \right| \chi_{i,n} C_{i,0} \rangle} \Gamma_i(t - T_R) & \text{if } t > T_R\,. \end{cases}$$

Quasi-statics

• Point kinetic calculation on the time interval ΔT_ϕ;

• At time T_ϕ, the factorized forms of the neutron density and delayed emissions are introduced in the spatial equations:

$$\begin{cases} A(t)\dfrac{\partial\varphi}{\partial t} + \varphi\dot{A} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right]\varphi A + \displaystyle\sum_{i=1}^{R} G_i e_i + S \\ \dfrac{1}{\lambda_i}\dfrac{\partial e_i}{\partial t}G_i + \dfrac{1}{\lambda_i}e_i\dot{G}_i + \dfrac{1}{\lambda_i}\boldsymbol{\nabla}\cdot(\mathbf{u}e_i)\,G_i = \\ \qquad \hat{\mathrm{M}}_i\varphi A - e_i G_i, \\ \qquad\qquad i = 1, 2, ..., R, \end{cases}$$

After time discretization the equations take the form:

$$\begin{cases} \dfrac{\varphi^{n+1}-\varphi^{n}}{\Delta T_\phi} + \varphi^{n+1} \left.\dfrac{\dot{A}}{A}\right|_{T_\phi} = \left[\hat{\mathrm{L}} + \hat{\mathrm{M}}_p\right] \varphi^{n+1} + \\ \qquad \displaystyle\sum_{i=1}^{R} e_i^{n+1} \left.\frac{G_i}{A}\right|_{T_\phi} + \frac{S}{A|_{T_\phi}} \\ \\ \dfrac{1}{\lambda_i} \dfrac{e_i^{n+1}-e_i^{n}}{\Delta T_\phi} + \dfrac{1}{\lambda_i} e_i^{n+1} \left.\dfrac{\dot{G}_i}{G_i}\right|_{T_\phi} + \dfrac{1}{\lambda_i} \boldsymbol{\nabla} \cdot \left(\mathbf{u} e_i^{n+1}\right) = \\ \qquad -\lambda_i C_i^{n+1} + \beta_i \displaystyle\sum_g \nu\Sigma_{fg} \phi_g^{n+1} \left.\frac{P}{G_i}\right|_{T_\phi} \\ \\ \qquad\qquad i = 1, \ldots,\ R \end{cases}$$

Iterations on the values of the derivatives of the amplitude, to fulfill the normalization conditions.

Typical Results (multigroup diffusion model, 2 groups)