Inverse problem in neutron kinetics

In general: reconstruct system parameters from some knowledge (experimental) of the system response

$\Longrightarrow$ **measurement of reactivity**

$\Longrightarrow$ Application to subcritical experiments

Pulsed experiments

The source is pulsed; frequency is in the kHz range and effective

system lifetime is shorter than one microsecond

$\Rightarrow$ pulses are isolated

$\Rightarrow$ the information is retrieved from the response to a single pulse

Scope: reconstruct reactivity from flux measurements

use of integrals of the power (energy release) is appropriate for experimental result interpretation

1. Obtain point kinetic solution for a pulsed subcritical system
2. Derive a simple deterministic formula to reconstruct reactivity (subcriticality level) from the response to the pulse
3. Analyse space and spectrum role in the interpretation of measurements and study the limits of the point model
4. Study the effect of experimental uncertainties

Analytical solution of point kinetic system of equations

Point kinetic equations

$$\begin{cases} \dfrac{dP(t)}{dt} = \dfrac{\rho(t)-\bar{\beta}}{l}P(t) + \displaystyle\sum_{i=1}^{6}\lambda_i\bar{C}_i(t) + \bar{S}(t) \\ \dfrac{d\bar{C}_i(t)}{dt} = \dfrac{\bar{\beta}_i}{l}P(t) - \lambda_i\bar{C}_i(t),\ i = 1, ..., 6 \end{cases}$$

+ initial conditions for P and $\bar{C}_i$

Assuming a constant reactivity, it is possible to obtain a fully analytical solution for the response to a pulse

$$P(t) = \sum_{j=1}^{7} \frac{1}{< e_j | e_j >} \times \left[\left(P(0) + \sum_{i=1}^{7} \frac{\lambda_i \bar{C}_i(0)}{\omega_j + \lambda_i} \right) e^{\omega_j t} + \int_0^t S(t') e^{\omega_j (t-t')} dt' \right]$$

time constants ω_j are eigenvalues of matrix (solutions of inhour equation)

$$\mathbb{A} = \begin{pmatrix} \frac{\rho - \beta}{l} & \lambda_1 & \ldots & \lambda_6 \\ \frac{\beta_1}{l} & -\lambda_1 & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\beta_6}{l} & 0 & \ldots & -\lambda_6 \end{pmatrix}$$

direct and adjoint eigenvectors:

$$|e_j >= \left(1, \frac{\beta_1/l}{\omega_j + \lambda_1}, ..., \frac{\beta_6/l}{\omega_j + \lambda_6}\right)^t,$$
$$< e_j| = \left(1, \frac{\lambda_1}{\omega_j + \lambda_1}, ..., \frac{\lambda_1}{\omega_j + \lambda_6}\right)$$

very complicate relationship between reactivity (which determines eigenvalues and eigenvectors) and the system power

$\Longrightarrow$ features of the point model

$\star$ ω_j: six of them are close to and approach each $-\lambda_i$ as subcriticality increases; the seventh one, ω_7, is much larger and negative and it determines the prompt response of the neutron population connected to the inverse of the prompt lifetime

$$P(t) = \sum_{j=1}^{7} \frac{1}{< e_j | e_j >} \times \left[\left(P(0) + \sum_{i=1}^{7} \frac{\lambda_i \bar{C}_i(0)}{\omega_j + \lambda_i} \right) e^{\omega_j t} + \frac{\tilde{S}}{\omega_j} \left(e^{\omega_j t} - 1 \right) \right] \quad \text{for } t \in [0, \tau],$$

$$P(t) = \sum_{j=1}^{7} \frac{1}{< e_j | e_j >} \left[\left(P(0) + \sum_{i=1}^{7} \frac{\lambda_i \bar{C}_i(0)}{\omega_j + \lambda_i} \right) e^{\omega_j t} + \frac{\tilde{S}}{\omega_j} \left(e^{\omega_j t} - e^{\omega_j (t - \tau)} \right) \right], \qquad \text{for } t \in [\tau, T]$$

$\star$ ω_7 can be approximated in a wide range of values of the kinetic parameters and for largely subcritical systems by $(\rho - \beta)/l$ (also, reduced role of delayed emissions for subcritical systems)

$\star$ the prompt response to the pulse is almost fully determined by ω_7

The exact value of the integral I of the power for an isolated pulse can be written as:

$$I = \int_0^T P(t)dt = \sum_{i=1}^{7} \frac{\tilde{S}}{< e_j|e_j >} \times \left[-\frac{\tau}{\omega_i} + \frac{1}{\omega_i^2} e^{\omega_i T}(1 - e^{-\omega_i \tau}) \right]$$

Approximation of the energy release by the prompt assumption:

$$I \cong \frac{\tilde{S}}{< e_7|e_7 >} \left[-\frac{\tau}{\omega_7} + \frac{1}{\omega_7^2} e^{\omega_7 T}(1 - e^{-\omega_7 \tau}) \right].$$

Since the first term in the above formula is predominant for small values $\hat{t}$ of time, the power can be approximated as

$$P(\hat{t}) \simeq \frac{1}{\omega_7} \frac{\tilde{S}}{< e_7|e_7 >} (e^{\omega_7 \hat{t}} - 1)$$

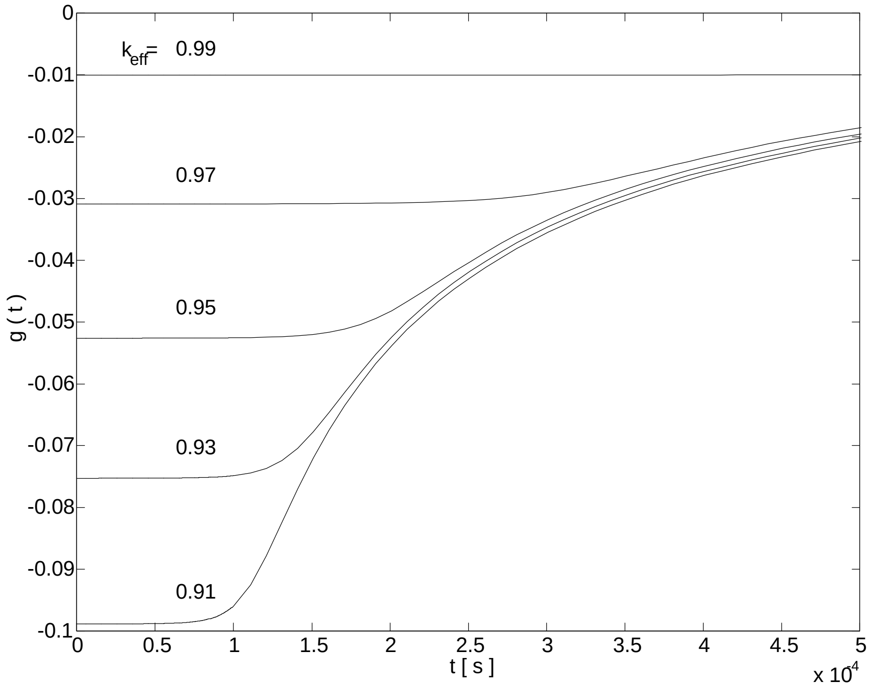
and the energy released in the pulse can be further approximated as:

$$I \cong -\frac{\tilde{S}}{< e_7|e_7 >}\frac{l\tau}{\rho - \beta}.$$

The effective source can be eliminated and the reactivity isolated as

$$\hat{\rho} = \hat{\rho}(\hat{t}) = \beta + \frac{l}{\hat{t}}\log\left(1 - \frac{P(\hat{t})\tau}{I}\right)$$

If $P(\hat{t})$ and I are exactly taken from the point model, $|\hat{\rho} - \rho|$ has a minimum with respect to $\hat{t}$, which gives the *best* approximation $\hat{\rho}$ for ρ



k_eff= 0.99
0.97
0.95
0.93
0.91
g (t)
t [s]
x 10^4
0
-0.01
-0.02
-0.03
-0.04
-0.05
-0.06
-0.07
-0.08
-0.09
-0.1
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5

Alternatively, a straight inverse method can be applied for the point model:

$$\rho = \beta + l\frac{d[\ln P(t)]}{dt} + \int\limits_{0}^{\infty} \left(\sum_{i=1}^{6} \lambda_i \beta_i e^{-\lambda_i u} \right) \times \frac{P(t-u)}{P(t)} du + l\frac{\tilde{S}}{P(t)}$$

in discretized form at each measurement instant t_i:

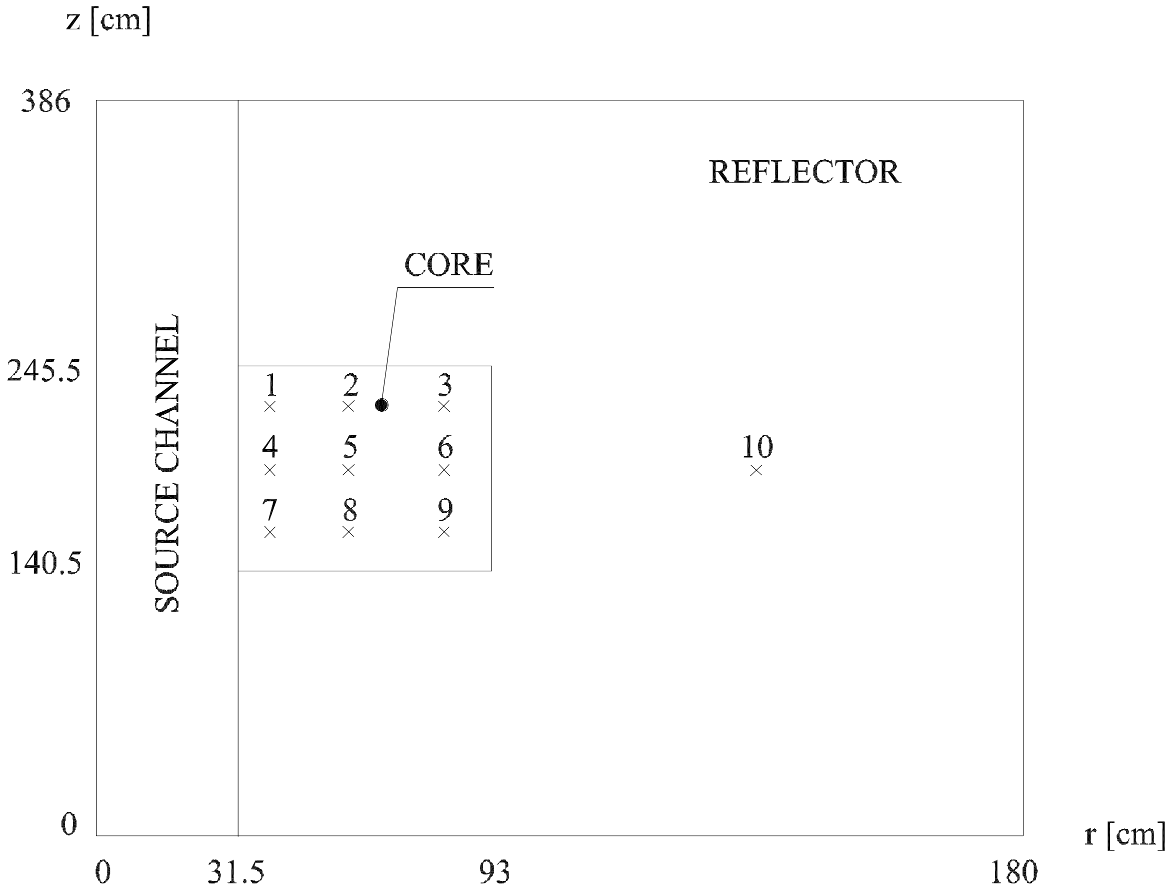
$$\rho(t_i) = \beta + l\frac{\ln P(t_{i+1}) - \ln P(t_i)}{t_{i+1} - t_i} + + \sum_{j=1}^{i-1} \left(\sum_{i=1}^{6} \lambda_i \beta_i e^{-\lambda_i t_j} \right) \times \frac{P(t_i - t_j)}{P(t_i)} \left(t_{j+1} - t_j \right) + l\frac{x_1}{P(t_i)}$$

RESULTS

Experiments are simulated by spatial code

$P(t)$ is generated by fully spatial numerical calculations

Verification of the adequacy of the approximation for a point reactor ($\beta = 0.0021$, $l = 1\ \mu s$, $\tau = 10\ ms$): differences of few pcm and extended minimum zone



1.Schematics of the system.

Analysis of the full power of the system.

k_{eff}	$\hat{k}_{eff}$	Δ (pcm)
0.97075	0.97144	+72
0.94500	0.94556	+60

The accuracy of the estimations by the point approximate formula depends strongly upon the point where the measurement is taken (the true system is not point-like), while the use of the total power yields accurate results

The deviation from the point behavior at short times is due to space effects; later the system departs from prompt behavior

To interpret local power measurements:

- Modal methods
- Multipoint models
- Reconstruction of total power

Information on total power can be constructed by a weighted sum of local measurements

$\Rightarrow$ volume weighting

$\Rightarrow$ importance weighting

Analysis of local power signals (k_{eff}=0.97075).

position	$\hat{k}_{eff}$	Δ (pcm)
4	no min	-
5	no min	-
9	0.97138	+66
3	0.97166	+94

Analysis of local power signals, volume weighting (k_{eff}=0.97075).

positions	$\hat{k}_{eff}$	Δ (pcm)
4, 5, 6	no min	-
1, 4, 7	no min	-
7, 8, 9	0.96952	+23
2, 5, 8	0.96758	-326
1, 2, 3	0.97046	-30
3, 6, 9	0.97143	+71
1 thru 9	0.96908	-172

Analysis of local power signals, importance weighting (k_{eff}=0.97075).

positions	$\hat{k}_{eff}$	Δ (pcm)
5, 6, 10	0.97097	+23
5, 10	0.97064	-11
6, 10	0.97159	+87

Interpret normal randomly distributed signals around the simulated value

Use information from the sequence of pulses

Effect of experimental uncertainties, no interpolation.

σ %	$\hat{k}_{eff,\max}$	$\hat{k}_{eff,\min}$	mean $\hat{k}_{eff}$	$\hat{\sigma}$	Δ (pcm
0.1	0.97108	0.94334	0.96695	0.3	+391
1	0.97111	0.92556	0.96326	0.6	+771
5	0.97337	0.88664	0.95871	0.9	+1240
10	0.97300	0.89706	0.95669	1.0	+1448

Effect of experimental uncertainties, third-degree polynomial interpolation.

σ %	$\hat{k}_{eff,\max}$	$\hat{k}_{eff,\min}$	mean $\hat{k}_{eff}$	$\hat{\sigma}$	Δ (pcm
0.1	0.97500	0.96732	0.97074	0.05	+1
1	0.98544	0.96478	0.96987	0.1	+90
5	0.99156	0.95845	0.96916	0.2	+164
10	0.99529	0.95461	0.96900	0.3	+180

Average value of the effective multiplication constant obtained by interpreting a sequence of 100 pulses with different uncertainties of the power measurement. Energy release method

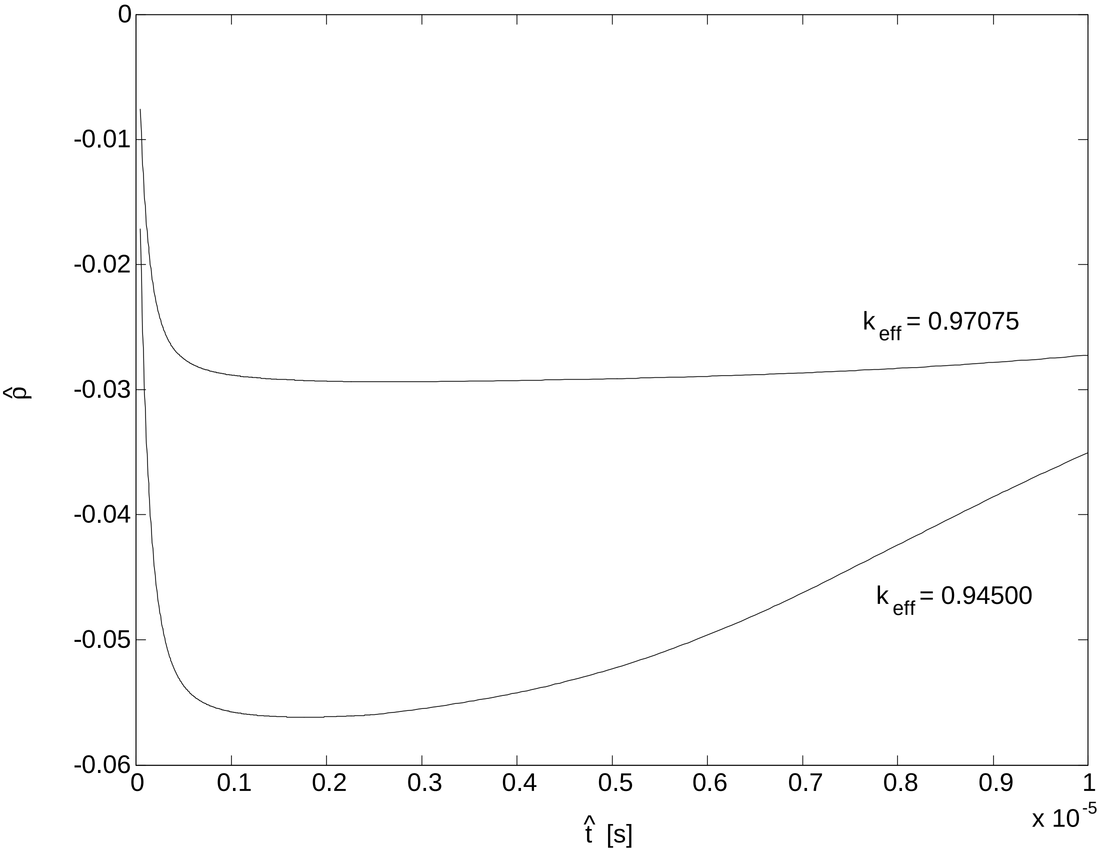
ϵ_P (%)	k	error (pcm)	σ_ρ ($\times 10^{-3}$)
1	0.97116	+42	0.14
5	0.97126	+52	0.74
10	0.97151	+78	1.54
20	0.97223	+152	3.31

Average value of the effective multiplication constant obtained by interpreting different sequences of 100 pulses with 10 % uncertainty on the power measurement. Energy release method

sequence	k	error (pcm)	σ_ρ ($\times 10^{-3}$)
1	0.97151	+78	1.54
2	0.97151	+78	1.54
3	0.97154	+81	1.57
4	0.97114	+40	1.46

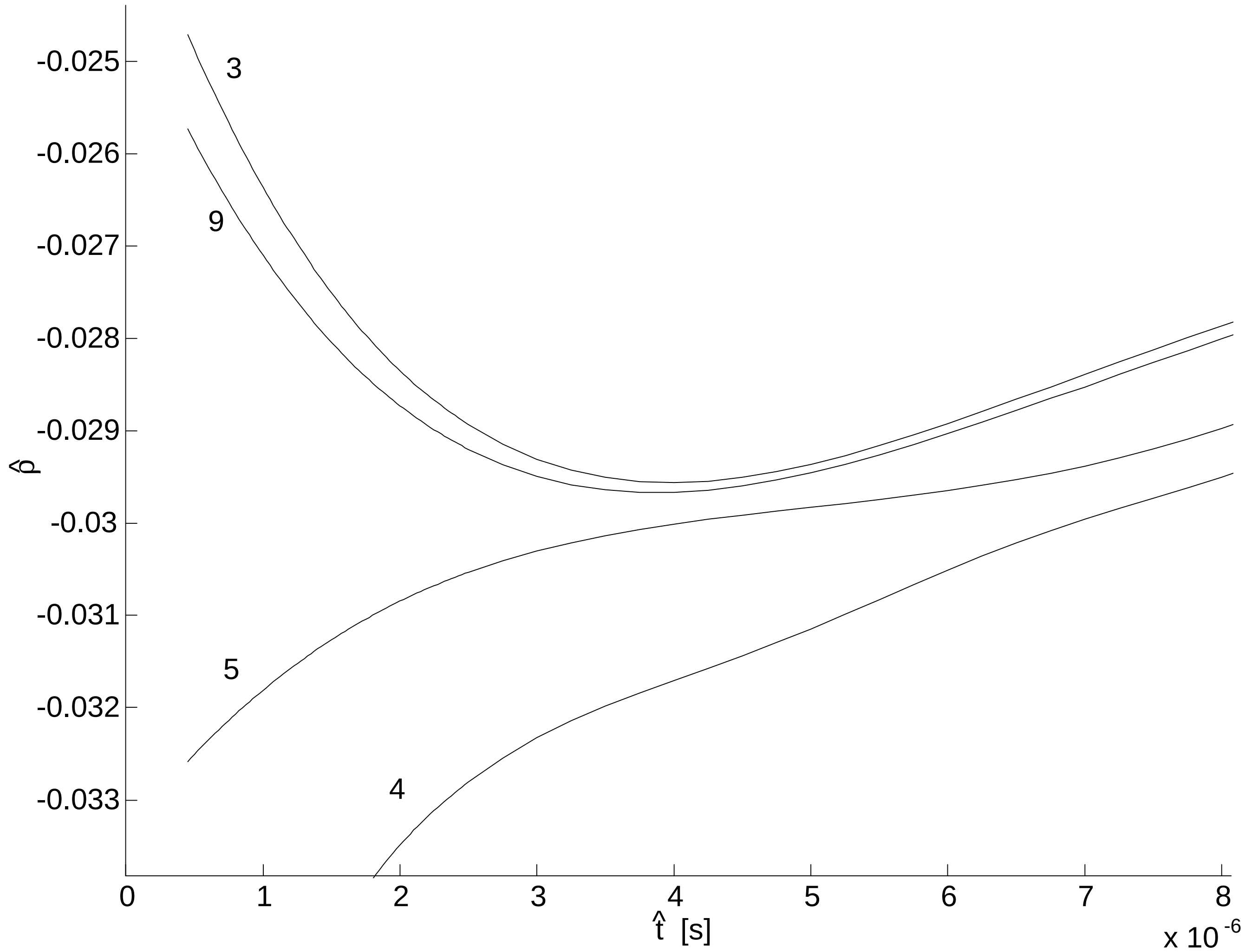
Average value of the effective multiplication constant obtained by interpreting different sequences of 100 pulses with differen uncertainties on the power measurement. Inverse kinetics method.

ϵ_P (%)	k	error (pcm)	σ_ρ ($\times 10^{-3}$)
1	0.97129	+54	0.30
5	0.97141	+66	1.51
10	0.97144	+69	3.09
20	0.97053	-22	6.75



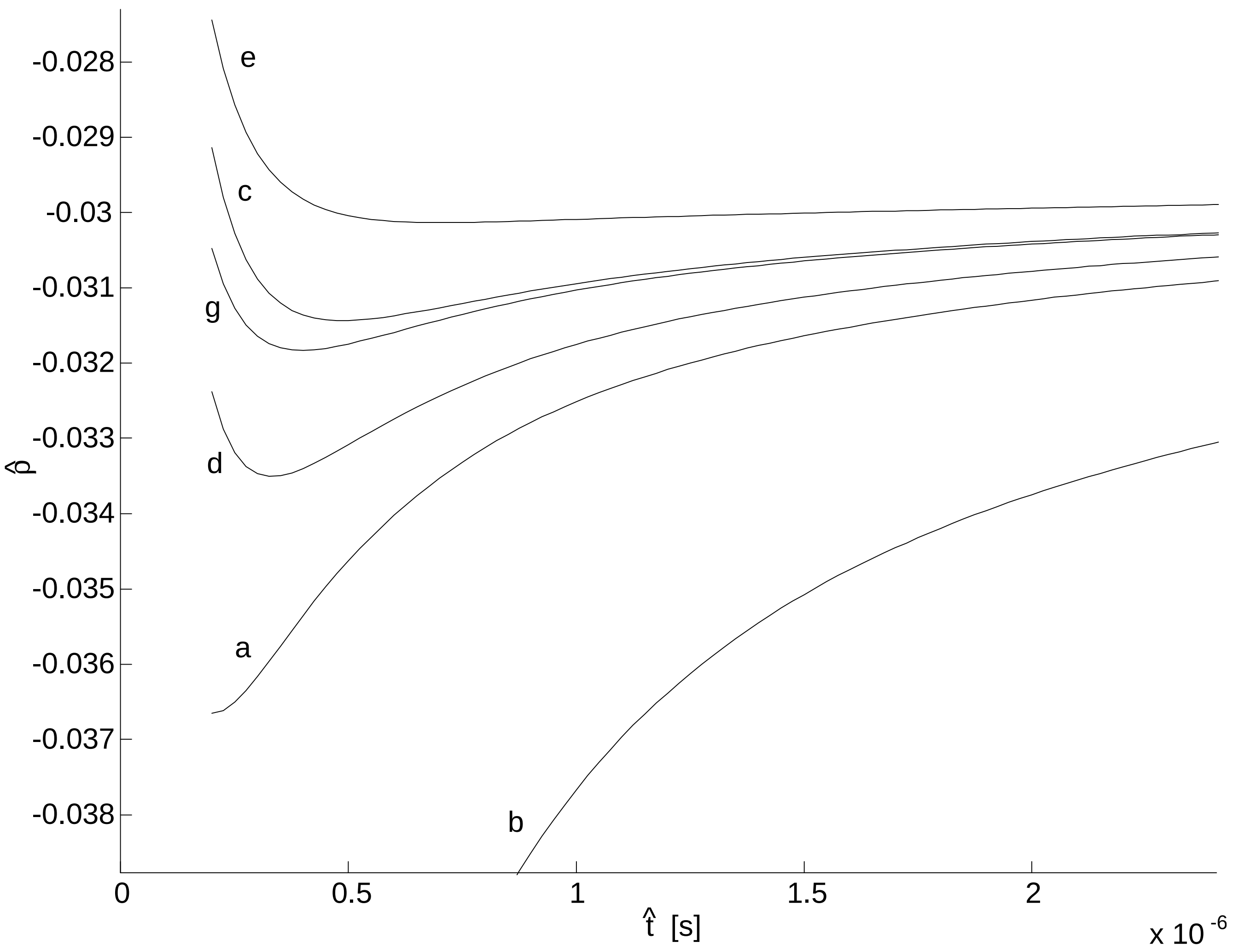
2.

Reactivity interpretation through the total power in systems having different subcriticality levels.



3.

Interpretation of local flux signals for a system caracterized by $k_{eff} = 0.97075$. Numbers indicate the position of the flux measurement.



4.

Interpretation of reconstructed power by volume weighting of point measurements. Points used to produce the curves: $a \equiv (4, 5, 6)$; $b \equiv (1, 4, 7)$; $c \equiv (7, 8, 9)$; $d \equiv (2, 5, 8)$; $e \equiv (1, 2, 3)$; $g \equiv 1$ thru 9.