

Fluid-fuel systems

Summary of lecture

- ▶ General model
- ▶ Discussion of dynamic effects of fuel motion
- ▶ Derivation of consistent point kinetics
- ▶ Quasi-static scheme

The neutron kinetics of circulating-fuel reactors

Balance equations for neutrons:

$$\left\{ \begin{array}{l} \frac{\partial n(\mathbf{r}, E, \Omega, t)}{\partial t} = \left[\hat{L}(t) + \hat{M}_p(t) \right] n(\mathbf{r}, E, \Omega, t) + \\ \sum_{i=1}^R \mathcal{E}_i(\mathbf{r}, E, t) + S(\mathbf{r}, E, \Omega, t) \\ \\ \frac{1}{\lambda_i} \frac{\partial \mathcal{E}_i(\mathbf{r}, E, t)}{\partial t} + \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u} \mathcal{E}_i(\mathbf{r}, E, t)) = \\ \hat{M}_i(t) n(\mathbf{r}, E, \Omega, t) - \mathcal{E}_i(\mathbf{r}, E, t), \\ \\ i = 1, 2, \dots, R, \end{array} \right.$$

$$\mathcal{E}_i(\mathbf{r}, E, t) = \lambda_i C_i(\mathbf{r}, t) \frac{\chi_i(E)}{4\pi}.$$

In the equations for precursors a convective term appears due to fuel motion, and appropriate boundary conditions must then be introduced:

$$\mathcal{E}_i(\mathbf{r}, E, t) \mathbf{u}(\mathbf{r}) \cdot (-\mathbf{n}) =$$

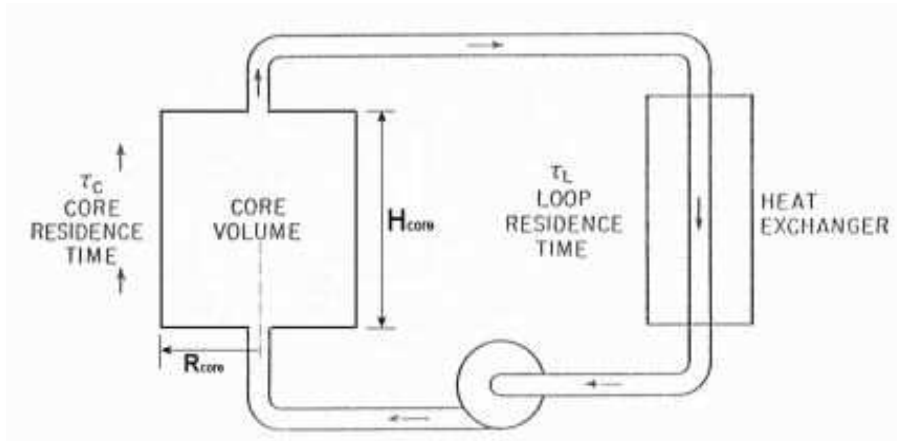
$$\int_{\mathcal{A}_{out}} \mathcal{E}_i(\mathbf{r}', E, t - \tau(\mathbf{r}' \rightarrow \mathbf{r})) e^{-\lambda_i \tau(\mathbf{r}' \rightarrow \mathbf{r})} \times$$

$$\mathbf{u}(\mathbf{r}') \cdot \mathbf{n}' \mathfrak{F}(\mathbf{r}' \rightarrow \mathbf{r}) d\mathcal{A}',$$

$$\mathbf{r} \in \mathcal{A}_{in}.$$

Fission products are moved through and outside the core by the motion of the fissile material.

Geometrical structure of a circulating-fuel reactor



Multigroup diffusion model in cylindrical geometry + slug flow

In slug-flow conditions the velocity field is maintained by externally-driven devices and is one-dimensional (axial):

$$\nabla \cdot (C_i \mathbf{u}) = \frac{\partial}{\partial z} (u C_i)$$

$$\tau_c = \frac{H_{core}}{\bar{u}_{core}}$$

$$\tau_L = \frac{L_{loop}}{\bar{u}_{loop}}$$

$$\frac{\bar{u}_{core}}{\bar{u}_{loop}} = \frac{A_{loop}}{A_{core}}$$

$$A_{core} = \pi R_{core}^2$$

$$\begin{aligned} \frac{1}{v_g} \frac{\partial \Phi_g}{\partial t} &= \nabla \cdot D_g \nabla \Phi_g - \Sigma_g \Phi_g + \\ &\sum_{g'} [\chi_{p,g} \nu \Sigma_{fg'} (1 - \beta) + \Sigma_{g' \rightarrow g}] \Phi_{g'} + \\ &S_g + \sum_i \lambda_i \chi_{i,g} C_i \end{aligned}$$

$$\frac{\partial C_i}{\partial t} = -\lambda_i C_i + \beta_i \sum_g \nu \Sigma_{fg} \Phi_g - \frac{\partial}{\partial z} (u C_i)$$

★ boundary conditions:

$$\begin{aligned} u(0) C_i(z = 0, r, t) &= \\ u(H) \frac{e^{-\lambda_i \tau_L}}{A_{core}} \int_{A_{core}} C_i(z = H, r, t - \tau_L) d\mathcal{A} \end{aligned}$$

Discussion of dynamic effects of motion

i) the delayed-precursor equations cannot be eliminated in steady-state configuration.

solid fuel: the concentrations of precursors can be expressed as functions of the fission term and substituted into the neutron balance equation

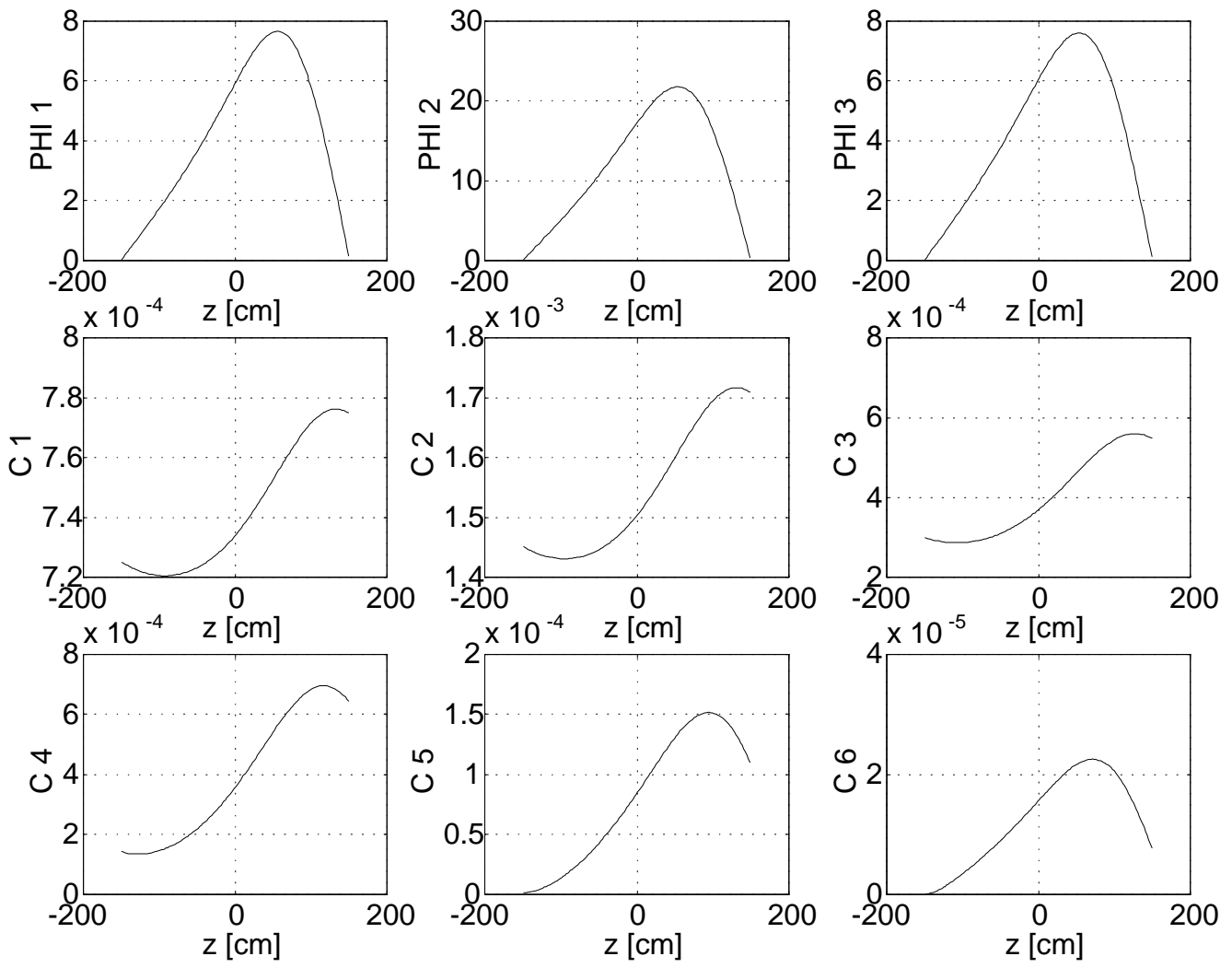
$$\mathcal{E}_i(\mathbf{r}, E) = \hat{M}_i n(\mathbf{r}, E, \Omega)$$

circulating fuel: the equations for precursors are still differential for the space variable and their concentrations can not be made explicit and substituted

$$\nabla \cdot (\mathbf{u}\mathcal{E}_i(\mathbf{r}, E)) = \lambda_i \hat{M}_i n(\mathbf{r}, E, \Omega) - \lambda_i \mathcal{E}_i(\mathbf{r}, E)$$

ii) the multiplication eigenvalue depends on delayed neutron and flow characteristics;

iii) the space distribution of delayed-precursors is not following the neutron distribution and is completely different from standard solid-fuel systems



iv) the role of delayed emissions is reduced by space-redistribution and external recirculation (reduction of effective β)

$$\beta_{i,eff} = \frac{\lambda_i \langle \Phi^+ | C_i \rangle}{\sum_j \lambda_j \langle \Phi^+ | C_j \rangle + (1 - \beta) \langle \Phi^+ | \nu \Sigma_f \Phi \rangle}$$

Ratio $\tilde{\beta}/\beta$ as a function of τ_L and k_{eff}

τ_L [s] \rightarrow	0	5	10	15	u [cm/s] \downarrow
$k_{eff} = 0.95009$	0.842	0.542	0.470	0.443	60
	0.835	0.422	0.363	0.332	100
$k_{eff} = 0.97048$	0.843	0.540	0.469	0.441	60
	0.834	0.420	0.353	0.330	100
$k_{eff} = 0.99028$	0.842	0.539	0.467	0.440	60
	0.833	0.419	0.352	0.329	100
$k_{eff} = 1.00001$	0.843	0.539	0.468	0.440	60
	0.834	0.420	0.352	0.329	100

.

v) factorization schemes should be applied to both neutrons and precursors

vi) point kinetics model needs a specific formulation

vii) the numerical solution can take advantage of the slower time-scale of the motion of the delayed precursors

Point kinetic model for circulating fuel systems (consistent with Henry factorization procedure)

⇒ the time-dependent balance equations for neutrons and delayed precursors are considered:

$$\left\{ \begin{array}{l} \frac{\partial n(\mathbf{r}, E, \Omega, t)}{\partial t} = \left[\hat{L}(t) + \hat{M}_p(t) \right] n(\mathbf{r}, E, \Omega, t) + \\ \sum_{i=1}^R \mathcal{E}_i(\mathbf{r}, E, t) + S(\mathbf{r}, E, \Omega, t) \\ \\ \frac{1}{\lambda_i} \frac{\partial \mathcal{E}_i(\mathbf{r}, E, t)}{\partial t} + \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u} \mathcal{E}_i(\mathbf{r}, E, t)) = \\ \hat{M}_i(t) n(\mathbf{r}, E, \Omega, t) - \mathcal{E}_i(\mathbf{r}, E, t), \\ i = 1, 2, \dots, R, \end{array} \right.$$

+initial and boundary condition

⇒ a reference configuration is introduced

$$\left\{ \begin{array}{l} 0 = \left[\hat{L}_0 + \hat{M}_{p,0} \right] N_0(\mathbf{r}, E, \Omega) + \\ \quad \sum_{i=1}^R \mathcal{E}_{i,0}(\mathbf{r}, E) + S_0(\mathbf{r}, E, \Omega) \\ \\ \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u}_0 \mathcal{E}_{i,0}(\mathbf{r}, E)) = \\ \quad \hat{M}_{i,0} N_0(\mathbf{r}, E, \Omega) - \mathcal{E}_{i,0}(\mathbf{r}, E), \\ \quad \quad \quad i = 1, 2, \dots, R, \end{array} \right.$$

+boundary conditions

⇒ a physically consistent definition of the neutron concentrations and delayed emissions importance n^\dagger and \mathcal{E}^\dagger is introduced;

⇒ the balance equations for the importance functions are shown to be the mathematical adjoint to the balance equations for neutrons and precursors, having defined the inner product as:

$$(\mathbf{w}^\dagger, \mathbf{w}) = \sum_{n=1}^{R+1} \langle w_n^\dagger | w_n \rangle = \sum_{n=1}^{R+1} \int_V dV \int_E dE \int_{4\pi} d\Omega \cdot w_n^\dagger w_n$$

where

$$\begin{aligned} \mathbf{w} &= (n, \mathcal{E}_1, \dots, \mathcal{E}_R)^t \\ \mathbf{w}^\dagger &= (n^\dagger, \mathcal{E}_1^\dagger, \dots, \mathcal{E}_R^\dagger) \end{aligned}$$

\Rightarrow The system of equations for the importance takes the form:

$$\left\{ \begin{array}{l} \left[\hat{L}_0^+ + \hat{M}_{p,0}^+ \right] N_0^+(\mathbf{r}, E, \Omega) + \sum_{i=1}^R \hat{M}_{i,0}^+ \mathcal{E}_{i,0}^+(\mathbf{r}, E) + \\ S_0^+(\mathbf{r}, E, \Omega) = 0, \\ \\ N_0^+(\mathbf{r}, E, \Omega) + \frac{1}{\lambda_i} \mathbf{u}_0 \cdot \nabla \left(\mathcal{E}_{i,0}^+(\mathbf{r}, E) \right) - \\ \mathcal{E}_{i,0}^+(\mathbf{r}, E) = 0, \quad i = 1, 2, \dots, R, \end{array} \right.$$

with boundary conditions:

$$\mathcal{E}_i^+(\mathbf{r}, E) = \int_{\mathcal{A}_{in}} \mathcal{E}_i^+(\mathbf{r}', E) e^{-\lambda_i \tau(\mathbf{r} \rightarrow \mathbf{r}')} \mathfrak{F}(\mathbf{r} \rightarrow \mathbf{r}') d\mathcal{A}',$$

$$\mathbf{r} \in \mathcal{A}_{out}.$$

⇒ Both flux and delayed emission distributions are factorized with an amplitude-shape formula:

$$n(\mathbf{r}, E, \Omega, t) = A(t)\varphi(\mathbf{r}, E, \Omega; t),$$

$$\mathcal{E}_i(\mathbf{r}, E, t) = G_i(t)e_i(\mathbf{r}, E; t) \quad i = 1, 2, \dots, R.$$

* The factorized formula is introduced into balance equations;

* A projection on the adjoint solution is taken;

* A normalization conditions is imposed on the shape functions to make the factorization unique:

$$\frac{d}{dt} \langle N_0^+ | \varphi \rangle = 0,$$

$$\frac{d}{dt} \langle \mathcal{E}_{i,0}^+ | e_i \rangle = 0, \quad i = 1, 2, \dots, R,$$

⇒ a point-like model is obtained

$$\left\{ \begin{array}{l} \Lambda_P \frac{dA}{dt} = \left(\rho_S - \tilde{\beta} \right) A + \sum_{i=1}^R \lambda_i \Gamma_i + \tilde{S} \\ \Lambda_i \frac{d\Gamma_i}{dt} = \left(\tilde{\beta}_i + \rho_i \right) A - \left(\lambda_i + \mu_{u,i} + \mu_{\xi,i} \right) \Gamma_i + \sigma_i \\ \\ i = 1, \dots, R \end{array} \right.$$

→ structure equivalent to the point kinetic model for solid-fuel systems;

→ kinetic parameters and effective delayed neutron functions Γ_i have different definitions;

→ unconventional terms:

ρ_i : perturbation of production;

$\mu_{u,i}$: perturbation of fluid velocity;

$\mu_{\xi,i}$: perturbation of recirculation

time;

- normalization factor

$$\mathcal{F} = \sum_{i=1}^R \sum_{n=1}^G \langle \Phi_n^\dagger | \chi_{i,n} \lambda_i C_{i,0} \rangle +$$

$$+ (1 - \beta) \sum_{n=1}^G \sum_{g=1}^G \langle \Phi_n^\dagger | \chi_n (\nu \Sigma_f)_{g,0} \Phi_{g,0} \rangle$$

- effective delayed neutron precursors

$$\Gamma_i = \frac{1}{\mathcal{F}} \sum_{n=1}^G \langle \Phi_n^\dagger | \chi_{i,n} C_{i,0} \rangle G_i(t)$$

- effective neutron source

$$\tilde{S} = \frac{1}{\mathcal{F}} \sum_{n=1}^G \langle \Phi_n^\dagger | S_n \rangle$$

- effective delayed neutron fractions

$$\tilde{\beta}_i = \frac{1}{\mathcal{F}} \sum_{n=1}^G \langle \Phi_n^\dagger | \chi_{i,n} \lambda_i C_{i,0} \rangle, \quad \tilde{\beta} = \sum_{i=1}^R \tilde{\beta}_i.$$

- reactivity $\rho_S = \rho_0 + \rho_{pert}$, where

$$\rho_0 = -\frac{1}{\mathcal{F}} \sum_{n=1}^G \left\langle S_n^\dagger \middle| \Phi_{n,0} \right\rangle$$

$$\rho_{pert} = \frac{1}{\mathcal{F}} \left\{ -\sum_{n=1}^G \left\langle \nabla \Phi_n^\dagger \middle| \delta D_n \nabla \Phi_{n,0} \right\rangle + \right.$$

$$+ \sum_{n=1}^G \left\langle \Phi_n^\dagger \middle| \sum_{g=1}^G (1 - \beta) \chi_n \delta (\nu \Sigma_f)_g \Phi_{g,0} \right\rangle +$$

$$- \sum_{n=1}^G \left\langle \Phi_n^\dagger \middle| \delta \Sigma_{R,n} \Phi_{n,0} \right\rangle +$$

$$\left. + \sum_{n=1}^G \sum_{g=1}^G \left\langle \Phi_n^\dagger \middle| \delta \Sigma_{g \rightarrow n} \Phi_{g,0} \right\rangle \right\}$$

- prompt neutron lifetime

$$\Lambda_P = \frac{1}{\mathcal{F}} \sum_{n=1}^G \left\langle \Phi_n^\dagger \middle| \frac{1}{v_n} \Phi_{n,0} \right\rangle$$

- generalized precursor lifetime

$$\Lambda_i = \frac{\langle C_i^\dagger | C_{i,0} \rangle}{G \sum_{n=1} \langle \Phi_n^\dagger | \chi_{i,n} C_{i,0} \rangle}; \quad i = 1, 2, \dots, R$$

- unconventional terms

$$\rho_i = \frac{1}{\mathcal{F}} \left\langle C_i^\dagger \left| \beta_i \sum_{g=1}^G \delta(\nu \Sigma_f)_g \Phi_{g,0} \right. \right\rangle$$

$$\mu_{u,i} = \frac{\langle C_i^\dagger | \frac{\partial}{\partial z} (\delta u C_{i,0}) \rangle}{G \sum_{n=1} \langle \Phi_n^\dagger | \chi_{i,n} C_{i,0} \rangle}$$

$$\mu_{\xi,i} = \frac{u_0(H) \int_{\mathcal{A}_{core}} C_i^\dagger(H) C_{i,0}(r, H) d\mathcal{A}}{G \sum_{n=1} \langle \Phi_n^\dagger | \chi_{i,n} C_{i,0} \rangle}$$

- apparent precursor source

$$\Xi = \frac{1}{\mathcal{A}_{core}} u_0(H) (\xi_{i,0} + \delta\xi_i) \times \int_{\mathcal{A}_{core}} C_i^\dagger(r, 0) d\mathcal{A} \int_{\mathcal{A}_{core}} C_{i,0}(r, H)$$

$$\sigma_i = \begin{cases} \frac{\Xi}{G} \Gamma_{i,0}, \\ \sum_{n=1}^G \langle \Phi_n^\dagger | \chi_{i,n} C_{i,0} \rangle \\ \text{if } t \leq T_R, \\ \\ \frac{\Xi}{G} \Gamma_i(t - T_R) \\ \sum_{n=1}^G \langle \Phi_n^\dagger | \chi_{i,n} C_{i,0} \rangle \\ \text{if } t > T_R. \end{cases}$$

Quasi-statics

- Point kinetic calculation on the time interval ΔT_ϕ ;
- At time T_ϕ , the factorized forms of the neutron density and delayed emissions are introduced in the spatial equations:

$$\left\{ \begin{array}{l} A(t) \frac{\partial \varphi}{\partial t} + \varphi \dot{A} = \left[\hat{L} + \hat{M}_p \right] \varphi A + \sum_{i=1}^R G_i e_i + S \\ \frac{1}{\lambda_i} \frac{\partial e_i}{\partial t} G_i + \frac{1}{\lambda_i} e_i \dot{G}_i + \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u} e_i) G_i = \\ \hat{M}_i \varphi A - e_i G_i, \\ \\ i = 1, 2, \dots, R, \end{array} \right.$$

After time discretization the equations take the form:

$$\left\{ \begin{aligned}
 & \frac{\varphi^{n+1} - \varphi^n}{\Delta T_\phi} + \varphi^{n+1} \frac{\dot{A}}{A} \Big|_{T_\phi} = [\hat{L} + \hat{M}_p] \varphi^{n+1} + \\
 & \quad \sum_{i=1}^R e_i^{n+1} \frac{G_i}{A} \Big|_{T_\phi} + \frac{S}{A} \Big|_{T_\phi} \\
 & \frac{1}{\lambda_i} \frac{e_i^{n+1} - e_i^n}{\Delta T_\phi} + \frac{1}{\lambda_i} e_i^{n+1} \frac{\dot{G}_i}{G_i} \Big|_{T_\phi} + \frac{1}{\lambda_i} \nabla \cdot (\mathbf{u} e_i^{n+1}) = \\
 & \quad -\lambda_i C_i^{n+1} + \beta_i \sum_g \nu_{\Sigma fg} \phi_g^{n+1} \frac{P}{G_i} \Big|_{T_\phi} \\
 & \quad \quad \quad i = 1, \dots, R
 \end{aligned} \right.$$

Iterations on the values of the derivatives of the amplitude, to fulfill the normalization conditions.

Typical Results (multigroup diffusion model, 2 groups)

