A REDUNDANT, 6-DOF PARALLEL MANIPULATOR STRUCTURE
WITH IMPROVED WORKSPACE AND DEXTERTY

R. S. Stoughton
R. Salerno
S. Canfield
C. Reinholtz

August 1994

Presented at the
5th International Symposium on Robotics & Manufacturing
Conference
August 15-17, 1994
Maui, Hawaii

Prepared for
the U.S. Department of Energy
under Contract DE-AC06-76RLO 1830

Pacific Northwest Laboratory
Richland, Washington 99352

DISCLAIMER

This report was prepared as an account of work sponsored by an agency of the United States Government. Neither the United States Government nor any agency thereof, nor any of their employees, makes any warranty, express or implied, or assumes any legal liability or responsibility for the accuracy, completeness, or usefulness of any information, apparatus, product, or process disclosed, or represents that its use would not infringe privately owned rights. Reference herein to any specific commercial product, process, or service by trade name, trademark, manufacturer, or otherwise does not necessarily constitute or imply its endorsement, recommendation, or favoring by the United States Government or any agency thereof. The views and opinions of authors expressed herein do not necessarily state or reflect those of the United States Government or any agency thereof.
DISCLAIMER

Portions of this document may be illegible in electronic image products. Images are produced from the best available original document.
A Redundant, 6-DOF Parallel Manipulator Structure with Improved Workspace and Dexterity

Robert S. Stoughton  
Pacific Northwest Laboratory

Robert Salerno, Steve Canfield and Charles Reinholtz  
Virginia Polytechnic Institute

Abstract

This paper presents a novel manipulator structure which combines two known parallel manipulator structures - a Stewart Platform (SP), and a double octahedral Variable Geometry Truss (VGT). The combined VGT+SP structure is redundant, using nine actuators to realize six-DOF motion. Combining the two structures allows the translational and orientational workspaces of the two individual structures to sum together to a much larger workspace than is generally achievable with parallel manipulator structures. In addition, the VGT portion of the structure allows the configuration of the Stewart Platform to be changed “on the fly” from one with a large workspace to one with high dexterity. A useful application of this structure is at the distal end of a truss-based manipulator, where it can serve as a dexterous wrist while preserving an internal passageway for cabling and/or conveyance systems.

Introduction

Parallel actuated manipulators have received considerable attention in the robotics literature. Almost 30 years ago, Stewart [1] introduced a parallel actuated manipulator with six degrees of freedom (DOF). This manipulator structure, known as the Stewart Platform (rightmost in Figure 1), consists of six identical legs, each including a prismatic actuator, a universal (UV) or Hooke joint connecting the leg to a base platform, and a spherical joint connecting the leg to a second mobile platform. The SP has received much attention from researchers (e.g. [2]) as it is relatively simple, yet possesses recognized advantages of parallel manipulators including a large payload capacity, high stiffness and good positioning accuracy. The SP also suffers from the principal disadvantage of parallel manipulator structures - a rather limited workspace compared with serial manipulators.

Another type of parallel manipulator structure is based on modifying statically determinant truss structures to include prismatic actuators in some of the members. These extensible members allow the geometry of the truss to be changed in a precise, controllable manner, and hence, are referred to as Variable Geometry Trusses (VGTs). A number of VGT geometries have been studied in the past [4,5]. A particular VGT geometry which has received much attention is referred to as the Double Octahedral VGT (center in Figure 2). This VGT includes three actuators, and

---

1Operated for the U.S. Department of Energy by Battelle Memorial Institute under Contract DE-AC06-76RLO 1830.
is a three-DOF structure capable of extension along its main axis (z axis), and rotation about the x and y axes (equivalent to that of a controlled UV joint). For the remainder of this paper, "VGT" will refer to this particular VGT geometry.

The modular structure of the VGT allows any number of VGT units to be connected together to form a redundant manipulator capable of maneuvering around obstacles in a cluttered environment. As with a static truss, all of the members are loaded essentially in pure tension or compression. This allows strength and stiffness requirements to be met with a much lower weight design than is possible with traditional manipulator structures. The open framework of the VGT provides a protected passageway for end effector utilities, waste conveyance and cabling.

Each octahedral of the VGT actually incorporates a Stewart Platform structure. Replacing the distal set of six fixed links with six prismatic actuators results in the combined VGT+SP structure which is the subject of this paper (leftmost in Figure 1, and Figure 2). The VGT+SP structure maintains the open framework of the truss structure while adding a high degree of dexterity with a much larger range of motion than is achievable with other parallel structures.

**Kinematic Properties**

Parallel manipulator structures are typically optimized to emphasize dexterity or workspace volume, or some combination of the two. Dexterity can be thought of as the ability of a manipulator to arbitrarily change its position and orientation, or apply forces and torques in arbitrary directions. Workspace volume is often divided into a 3-D volume within which a manipulator's wrist may be positioned with a given orientation (total workspace) or while maintaining a given range of orientational motion (referred to as the dexterous workspace), along with the 3-D orientational range of motion of the manipulator's wrist at a given location. This breakdown of the 6-D workspace volume is less obvious for parallel manipulator structures (i.e., the orientational range of motion of the wrist is dependent on the location of the wrist) but is still useful.

It has been shown that for a Stewart Platform, changing the ratio of the base platform diameter to the mobile platform diameter changes the manipulator from one with a large workspace and poor dexterity to one with a small workspace and excellent dexterity [3]. Stewart Platform geometries emphasizing workspace and dexterity are shown in Figure 3 along with the dexterous workspaces of each design.

One of the principal advantages of the VGT+SP structure is that the VGT portion of the mechanism can actively change the geometry of the Stewart Platform to realize a highly dexterous geometry or a large workspace geometry as the task requires. Thus, the workspace and dexterity capabilities of the combined structure are far greater than a simple linear combination of the two. This change in the properties of the Stewart Platform are obtained when all three actuators of the VGT expand and contract equally so that their lengths remain equal. A still larger workspace is obtained when the VGT actuators move unequally so as to realize a bend angle within the VGT itself. Of particular importance when this structure is used as a dexterous "wrist" at the end of a truss-based manipulator, the VGT greatly increases the pitch and yaw orientational range of motion of the combined structure. Analysis of the actual workspace volume of the VGT+SP manipulator structure is currently under investigation by the authors and will be addressed in a future paper.

**VGT Kinematics** - The nine nodes of the VGT (and the VGT+SP) are identified in Figure 2. The position vector of each node is given by \( \mathbf{n}_i \). Each individual link is identified by the two nodes which define the link's end points, i.e., \( l_{ij} \) is the length of the link connecting nodes \( i \) and \( j \), and \( \hat{z}_{ij} \) is the unit vector in the direction of this link.

The three actuator velocities are related to the midplane node velocities by
\[
\left( \hat{n}_i - \hat{n}_j \right) \cdot \hat{z}_{ij} = i_j \quad \text{for } \{i,j\} = \{4,5\}, \{5,6\}, \{6,4\}
\]  

(1)

The closed-loop structure imposes a constraint that the velocity of any midplane node must be normal to each of the fixed links which connect that node to the base frame. Thus, only the magnitude of the midplane node velocities (\(v_i\)) is unknown while the direction (\(\hat{v}_i\)) is given by

\[
\hat{v}_i = \frac{\hat{z}_{ik} \times \hat{z}_{ji}}{\|\hat{z}_{ik} \times \hat{z}_{ji}\|} \quad \text{for } \{i,j,k\} = \{4,1,2\}, \{5,2,3\}, \{6,3,1\}
\]  

(2)

Substituting (2) into (1) and rearranging in matrix form yields the relationship between actuator velocities and midplane node velocities as

\[
J_{Mid}^{-1} \vec{v} = \hat{v}_{VGT}
\]  

(3)

where

\[
J_{Mid}^{-1} = \begin{bmatrix}
\hat{V}_4 \cdot \hat{z}_{45} & -\hat{V}_5 \cdot \hat{z}_{45} & 0 \\
0 & \hat{V}_5 \cdot \hat{z}_{56} & -\hat{V}_6 \cdot \hat{z}_{56} \\
-\hat{V}_4 \cdot \hat{z}_{64} & 0 & \hat{V}_6 \cdot \hat{z}_{64}
\end{bmatrix}, \quad \vec{v} = \begin{bmatrix} v_4 \\ v_5 \\ v_6 \end{bmatrix}, \quad \text{and } \hat{v}_{VGT} = \begin{bmatrix} i_{45} \\ i_{56} \end{bmatrix}
\]

The velocities of nodes 7, 8 and 9 (the top plane) may be expressed as a function of the linear (\(\vec{V}_T\)) and angular (\(\vec{\Omega}_T\)) velocities of the end frame \(\{T\}\) relative to the base frame as

\[
\hat{\dot{s}}_i = \vec{V}_T + \vec{\Omega}_T \times \vec{s}_i \quad \text{for } i = 7,8,9
\]  

(4)

where \(\vec{s}_i\) is the position of node \(i\) in the (moving) \(\{T\}\) frame. Each of the six links connecting the top plane and midplane are of fixed length, which yields a motion constraint of the form

\[
\left( \hat{n}_i - \hat{n}_j \right) \cdot \hat{z}_{ij} = 0 \quad \text{for } \{i,j\} = \{4,7\}, \{5,8\}, \{6,9\}, \{4,8\}, \{5,9\}, \{6,7\}
\]  

(5)

Substituting (4) into (5) results in

\[
\hat{\dot{s}}_i \cdot \vec{V}_T + \left( \hat{\dot{s}}_i \times \vec{z}_i \right) \cdot \vec{\Omega}_T = \left( \hat{\dot{s}}_i \cdot \hat{v}_i \right) v_i \quad \text{for } \{i,j\} \text{ as in (5)}
\]  

(6)

or,

\[
J_{Top}^{-1} \dot{x} = Q \begin{bmatrix} \vec{v} \\ \vec{\Omega}_T \end{bmatrix} \quad \text{where } J_{Top}^{-1} = \begin{bmatrix}
\hat{z}_{47}^T (\vec{s}_1 \times \hat{z}_{47})^T \\
\hat{z}_{48}^T (\vec{s}_8 \times \hat{z}_{48})^T \\
\hat{z}_{49}^T (\vec{s}_9 \times \hat{z}_{49})^T \\
\hat{z}_{57}^T (\vec{s}_5 \times \hat{z}_{57})^T \\
\hat{z}_{59}^T (\vec{s}_9 \times \hat{z}_{59})^T \\
\hat{z}_{67}^T (\vec{s}_7 \times \hat{z}_{67})^T
\end{bmatrix}, \quad \dot{x} = \begin{bmatrix} \vec{V}_T \\ \vec{\Omega}_T \end{bmatrix}
\]  

(7)

and \(Q = \text{diag}\{(\vec{z}_{47} \cdot \hat{V}_4), (\vec{z}_{58} \cdot \hat{V}_5), (\vec{z}_{49} \cdot \hat{V}_6), (\vec{z}_{48} \cdot \hat{V}_4), (\vec{z}_{59} \cdot \hat{V}_5), (\vec{z}_{67} \cdot \hat{V}_6)\}\)
Since $J^{-1}_{\text{Mid}}$ and $J^{-1}_{\text{Top}}$ are in general nonsingular, the overall Jacobian of the VGT is

$$\dot{x} = J_{VGT} \dot{t}_{VGT} \quad \text{where} \quad J_{VGT} = J_{\text{Top}} O \begin{bmatrix} J_{\text{Mid}} \end{bmatrix}$$  \hspace{1cm} (8)

**Stewart Platform Kinematics** - If the VGT actuators are fixed and SP actuators allowed to move, only the top plane nodes $(n_1, n_2, n_3)$ move - the midplane nodes $(n_4, n_5, n_6)$ remain stationary. Reexamining (4)-(7) results in the velocity relationships

$$J_{\text{Top}}^{-1} \dot{x} = \dot{t}_{SP} \Rightarrow \dot{x} = J_{\text{Top}} \dot{t}_{SP} \quad \text{where} \quad \dot{t}_{SP} = \begin{bmatrix} i_4 \ i_5 \ i_6 \ i_4 \ i_5 \ i_6 \end{bmatrix}^T$$  \hspace{1cm} (9)

and $\dot{x}$ is as in (7). This derivation is given in slightly different form in [2].

**Overall VGT+SP Jacobian Relationships** - The overall Jacobian relationships are found by combining the Stewart Platform and VGT Jacobian relationships as

$$J_{VGT+SP} = \begin{bmatrix} J_{\text{Top}} & J_{VGT} \end{bmatrix}$$

where $J_{VGT+SP}$ is defined by $\dot{x} = J_{VGT+SP} \dot{t}_{VGT+SP}$ and $\dot{t}_{VGT+SP} = \begin{bmatrix} i_{SP} \\ i_{VGT} \end{bmatrix}$.

**Conclusion**

A new parallel structure for a robotic manipulator was presented which combines two previously known parallel manipulator structures. The new structure is redundant, possessing nine actuators to realize six-DOF motion. The new structure has the capability of a greatly expanded workspace, both translational and orientational, versus previously known parallel manipulator structures. The unique combination of a Stewart Platform with a VGT allows the structure to realize optimum dexterity when working within a central subregion of its workspace without sacrificing its ability to span a much larger workspace when needed. The Jacobian relationships which relate actuator static forces and velocities to end frame forces and velocities were derived.

Future work will focus on quantifying the workspace of the VGT+SP structure, and on using the derived Jacobian equations to develop real time control algorithms which allow the manipulator to maintain optimum dexterity while moving along a given trajectory.

**Bibliography**


Figure 1 - The VGT+SP, VGT and Stewart Platform Structures.

Figure 2 - Node Numbering for the VGT+SP

Figure 3 - Stewart Platform Geometries Emphasizing Workspace and Dexterity.