EVALUATING THE PBMR MAIN POWER SYSTEM TURBO MACHINES WITH THE AID OF SENSITIVITY AND MONTE CARLO ANALYSES

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ABSTRACT: Sensitivity and Monte Carlo analyses are performed on the turbo machines of the PBMR Main Power System (MPS). The purpose of the study is twofold. Firstly, to highlight the MPS parameters that have the biggest influence on the turbo machines, as well as the extent of this influence. Secondly, the results are used to evaluate the present turbo machine design: Do the results fall within the present operational envelope or is a turbo machine design change required? The turbo machine parameters used as evaluating criteria are the turbo unit rotational speed, the compressor surge margins, the grid power produced and the net cycle efficiency. Four operating steady states, which encompass the plant operating envelope, are evaluated. The following tools are used: a proprietary fast solving network Computational Fluid Dynamics (CFD) code that solves conservation of mass, energy and momentum equations. In order to automate the process of solving thousands of steady states, additional code was developed in Matlab, which was also used to process the output data. This paper briefly describes the type and origin of the uncertainty parameters identified and how these parameters are varied. The application of variance distributions used, e.g. normal, triangular and uniform distributions, is also mentioned. The paper concludes with some key results, for example the list of most sensitive parameters identified and the fact that the results verify the robustness of the PBMR MPS is discussed.

KEYWORDS: PBMR, turbo machines, sensitivity analysis, Monte Carlo analysis

0. INTRODUCTION

The PBMR Project in South Africa utilizes a recuperative direct Brayton cycle with intercooling. Helium, the gas driving the turbines, also cools the reactor. This paper describes the analysis performed on the three-shaft concept. The three shafts are made up of the High-pressure Turbo Unit (HPTU), the Low-pressure Turbo Unit (LPTU) and the Power Turbine Generator (PTG). The HPTU and LPTU comprise free running shafts, with a turbine driving a compressor. The power turbine drives the generator.

Sensitivity analysis is performed on the Main Power System (MPS) to determine the most sensitive parameters influencing the MPS turbo machines. This is done to gain a greater understanding of the MPS. The Monte Carlo analysis gives an indication of what the expected variation in output parameters is. Should this variation fall outside the design envelope of the turbo machines, it would necessitate a redesign of sorts, or in the worst-case scenario, a change in operating philosophy.

1. THE STUDY

Sensitivity analysis as described in this paper means varying a single input parameter at a time and recording the response in output parameters. This is opposed to the Monte Carlo analysis where all the input parameters are varied simultaneously in a random fashion. The following **output parameters** were examined with regard to the MPS and its turbo machinery:

- The HPTU and LPTU rotational speed,
- The High-pressure Compressor (HPC) and Low-pressure Compressor (LPC) surge margins,
- The grid power produced, and
- The net cycle efficiency.

The turbo machinery supplier specifies a safe rotational speed region for the free running turbo units and a surge margin that is consistent with conventional gas turbine experience. Grid power and cycle efficiency is very important for the business case of PBMR, since these are the main selling points of the plant.

In total, 49 **input** parameters were varied. These parameters consisted mainly of the following:

- Loss factors of the turbo machinery intakes and diffusers,
- Leakage flows used for cooling,
- Turbo machine efficiencies,
- Turbo machine map accuracy,
- Heat exchanger parameters such as the heat transfer areas,
- Reactor parameters like the pressure drop and outlet temperature, and
- Manifold pressure and cooling water temperature.

Four operational **steady states** were selected as basis for the study. These steady states encompass the expected operational envelope of the PBMR plant. The steady states are the following:

- Full Power at Full Inventory, 900 °C Reactor Outlet Temperature (ROT)
- Full Power at 40% Inventory, 900 °C ROT
- Reduced Power at 100% Inventory, 750 °C ROT
- Reduced Power at 40% Inventory, 750°C ROT

The sensitivity and Monte Carlo analysis studies were carried out for each of these steady state conditions, and the results evaluated.

In order to perform thermo hydraulic analysis at PBMR, a proprietary, fast solving network CFD code was developed. This code is capable of solving the PBMR cycle by considering mass, energy and momentum conservation. In order to automate the process of solving several thousands of steady state runs, software was developed in Matlab to automatically call the thermo hydraulic code. Matlab is also used to process and analyse the results.

2. LIMITATIONS OF THE STUDY

The following limitations are imposed on the study:

- No external control was implemented while doing any of the steady state runs. This means that the reactor power was not limited to 400 MW, but was allowed to go over 400 MW. The maximum power reached in the Monte Carlo runs was in the order of 440 MW.
- All the parameters that were varied during both analyses were assumed to be capable of varying independently of each other. This is not necessarily true, especially considering the turbo-machine parameters. It is doubtful, for example, that it may be possible to improve the efficiency of the turbines and have complete flexibility regarding the flow area size of the machine, which is what the Monte-Carlo assumes. For the simulation, it was assumed that cross-coupling effects such as these have a non-material effect on the results.
- Effects such as reduction in turbine efficiency as a function of time (due to wear and tear) have not been considered.

3. THE ORIGIN OF UNCERTAINTY

Uncertainty originates from many sources: design, manufacturing, measuring, control, etc. The variables chosen and the variation employed, attempted to encompass all, or a combination of, the mentioned uncertainty sources. The following paragraphs elaborate on the chosen variation for the parameters mentioned.

Resistive losses: The MPS turbo machines have not been built, and until such a time when this has been done, uncertainty will exist as to the exact performance of the turbo machine intakes and diffusers. CFD is utilized by both the supplier and PBMR to check pressure drop and secondary loss coefficients of these important components. The secondary loss coefficients are used in equivalent pipe models in the MPS thermo-hydraulic model. The upper limit of the expected variation was chosen much further from the nominal value, as was the choice of lower limit. It was reasoned the pressure drop would more likely be higher than lower.

The exact reactor pressure drop will only be known once the reactor has been built and tested. For the purposes of this study, varying the length of the reactor creates the variation in pressure drop.

Leakage flow: Process helium cools the electromagnetic bearings that support the MPS turbo machines. This process gas is in essence a leakage flow bypassing the reactor. This leakage flow is supplied by the HPC and LPC, which use energy that could have done useful work. Again, the upper limit was chosen further from the nominal than the lower limit value, as there is always a chance that the required leakage might be more than anticipated.

Turbo machine efficiency: Turbo machine efficiency is a vital part in designing a high efficiency MPS. The supplier stipulates three efficiency values, with 10%, 50% and 90% risk. It is envisaged that the high efficiency value (90% risk) will be achieved in the production plants, when the machines have been proven in service. The turbo machine efficiency is varied within the band of 10% to 90% risk efficiency.

Turbo machine geometry: Due to manufacturing tolerances, there is always a difference between the designed turbo machine performance and the as-built performance. The main source of this variation is in the thickness of axial machine blades. Differences in blade thickness result in a difference in throat area that translates to a different mass flow at given pressure ratio. The turbo machinery supplier estimated the variation in throat area from past experience.

Heat exchanger variation: Heat exchangers also form an integral part of the MPS. The heat exchangers are designed for the plant design point (full power, full inventory). The heat exchangers are more efficient at low inventory, since they are oversized for these conditions. Thus, in order to get a visible variation in output, it was decided to vary the heat exchanger heat area between -50% and +50%.

Reactor outlet temperature: Reactor outlet temperature is the highest temperature in the thermodynamic cycle and is, therefore, an important parameter for net cycle efficiency. The reactor controllers, with feedback from the temperature sensors, control the outlet temperature, but process and sensor variation will cause variations around the nominal value of 900 °C. It is estimated that the controlled temperature can fluctuate in a band of ±15 °C around the set-point value. This variation is included in this study.

Cooling water temperature: The MPS cooling water connects to the ultimate heat sink through an intermediate heat exchanger. The cooling water temperature of the MPS heat exchangers will vary with the variation in temperature of the ultimate heat sink, which is in a band of ±3 °C. This variation is included in this study.

Manifold Pressure: The manifold pressure relates to the inventory in the MPS. A controller, that will also have a variation about its set point, controls manifold pressure. This expected variation of ±1% is also included in this study.

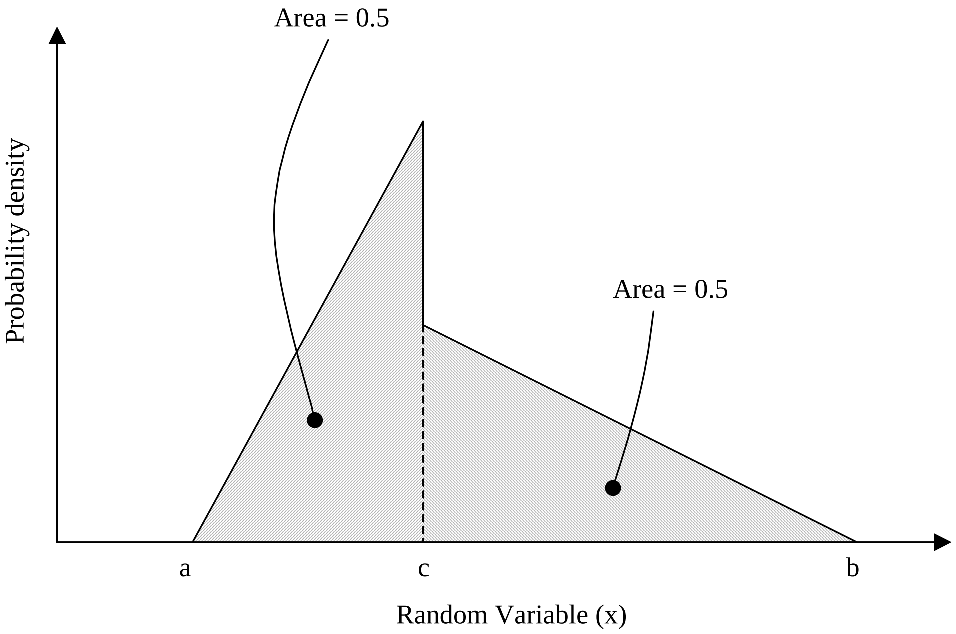
4. VARIANCE DISTRIBUTIONS

Three variance distributions were used for the Monte Carlo analysis. These are the double triangular distribution, normal distribution and uniform distribution. The double triangular distribution was derived for the purpose of this study, while the other two distributions are the standard variations as implemented in Matlab.

The double triangular distribution was derived to cater for the skew distributions used for most of the variable variances. As shown in FIGURE 1, the area of each triangle was said to be 50%, meaning that the random number generator had an equal chance of creating a value in either direction, be it smaller or larger than the nominal design value.

As for the normal and uniform distributions, the standard Matlab functions describing these distributions were used. The uniform distribution is the most conservative variation, since it gives equal chance to all values. The uniform distribution was used for the variation of the recuperator leakage as well as the reactor pressure drop, as these values will only be known exactly once these components have been tested.

The normal distribution was used as the variance of the variables that had symmetrical upper and lower limits. Mostly these variables are the ones actively being controlled by the control system, such as the reactor outlet temperature and the manifold pressure. The sea temperature at Koeberg where the demonstration plant is to be built also has a symmetrical variation throughout the year, and thus the variation in cooling water was also represented by the normal distribution.



Probability density function:

$p(x)=0$ for $x<a$

$p(x)=\frac{x-a}{(c-a)^2}$ for $a\le x\le c$

$p(x)=\frac{b-x}{(b-c)^2}$ for $c<x\le b$

$p(x)=0$ for $x>b$

Cumulative probability function:

$P(x)=0$ for $x<a$

$P(x)=\frac{1}{2}\left(\frac{x-a}{c-a}\right)^2$ for $a\le x\le c$

$P(x)=1-\frac{1}{2}\left(\frac{b-x}{b-c}\right)^2$ for $c<x\le b$

$P(x)=1$ for $x>b$

FIGURE 1. The shape and definition of the triangular probability density function.

5. RESULTS

5.1 SENSITIVITY ANALYSIS RESULTS

Remember that each variable was varied individually to find the sensitivity of the output parameters to the inputs. The inputs were varied in 10 equal increments from the lower to the upper limit. The inputs and outputs were then normalized by dividing with the nominal values. A straight line was fitted to the input-output relationship, using a least squares regression. The gradient (dy/dx) of this line is an indication of the percentage change in output for a 1% change in input and hence its sensitivity. Absolute gradient values were used when determining the sensitivity parameter lists.

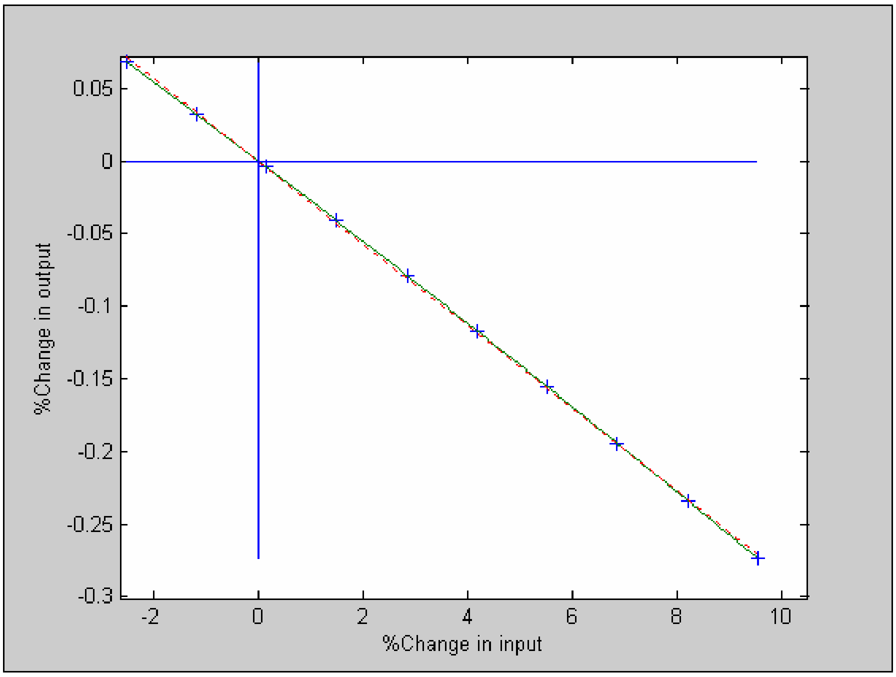


FIGURE 2. Typical Sensitivity Analysis Result.

Fitting a straight line to all the input-output variable relationships enables one to draw up tables for each of the four plant operating steady states that describe the sensitivity of each input parameter to the six output parameters. These tables can then be sorted to obtain a list from the most sensitive (largest gradient) to the least sensitive (smallest gradient) parameter and from this the top ten parameters are found for each output parameter.

In order to combine the six top ten lists for each operating steady state considered, different methods were followed which produced slightly different rankings. The following four methods were considered:

- Allocating a score of one to ten to each top ten position and then adding the scores together to form the combined list.
- Normalizing each top ten list by dividing by the maximum sensitivity (largest gradient) and then adding the normalised values to obtain the combined list.
- Simply adding the calculated gradient values of the respective parameters together to form the combined list.
- The variation in input parameter multiplied with the calculated gradient value and then these values were added to obtain the combined list.

The last method is the only method that considers the input variation. Keeping in mind that the input variations chosen were larger than expected real plant variations in almost all instances, the last method should give the most accurate assessment of the parameter sensitivities.

By combining the top ten lists as explained above, the first three methods revealed the same fourteen parameters as the most sensitive. The position of the parameters differed slightly on the three lists. The list of important parameters comprised of (not in order of precedence):

- Turbo machine performance map accuracy,
- Reactor outlet temperature,
- Cooling water temperature,
- Maximum pressure,
- Recuperator effectiveness.

By utilizing the fourth method, which also considers the input variation, the list of fourteen parameters grew to twenty-two. The additional parameters that came to the fore were:

- Reactor bypass leak,
- Reactor pressure drop,
- Compressor diffuser losses,
- Pre-cooler and intercooler effectiveness.

5.2 MONTE CARLO ANALYSIS RESULTS

The Monte Carlo analysis used the same input parameters, the same upper and lower variable limit and results focused on the HPTU and LPTU rotational speed, HPC and LPC surge margin, grid power and net cycle efficiency. The same four plant operating steady states that were used for the sensitivity analysis, were used.

A previous Monte Carlo analysis revealed that 1 000 runs is sufficient to obtain meaningful results. In order to verify that this is true, Monte Carlo runs comprising 250 and 2000 runs were also performed. The results are compared in TABLE 1 and

TABLE 2.

TABLE 1. Comparison of Cycle Efficiency for different number of Monte Carlo runs.

Runs	Nom cycle efficiency	Mean	Lower limit	Upper limit	10% value	90% value
250	40.5062	40.3997	37.9892	42.1769	39.3450	41.3875
1 000	40.5062	40.3948	37.4783	42.4378	39.4413	41.3191
2 000	40.5062	40.3671	37.4783	42.6122	39.4020	41.2900

TABLE 2. Percentage difference.

Runs	**Nom cycle efficiency**	**Mean**	**Lower limit**	**Upper limit**	**10% value**	**90% value**
250	0.000%	0.081%	1.363%	-1.022%	-0.145%	0.236%
1 000	0.000%	0.069%	0.000%	-0.409%	0.100%	0.070%
2 000	Base	Base	Base	Base	Base	Base

From TABLE 1 and TABLE 2 it can be seen that 1 000 runs is sufficient to obtain Monte Carlo convergence for the goal of our analysis. It would not make sense to do runs of 10000 given the assumptions made for the input variations.

The results of the Monte Carlo runs for the Full Power, Full Inventory steady state were as follows:

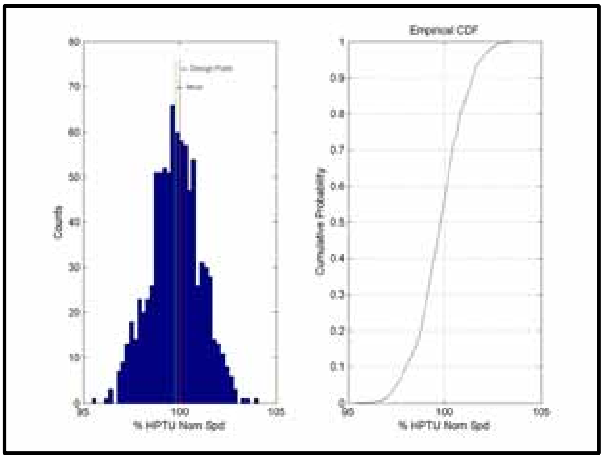


FIGURE 3. HPTU nominal rotational speed

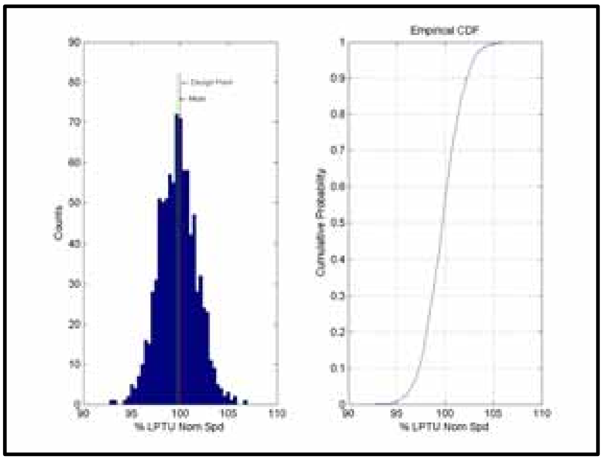


FIGURE 4. LPTU nominal rotational speed.

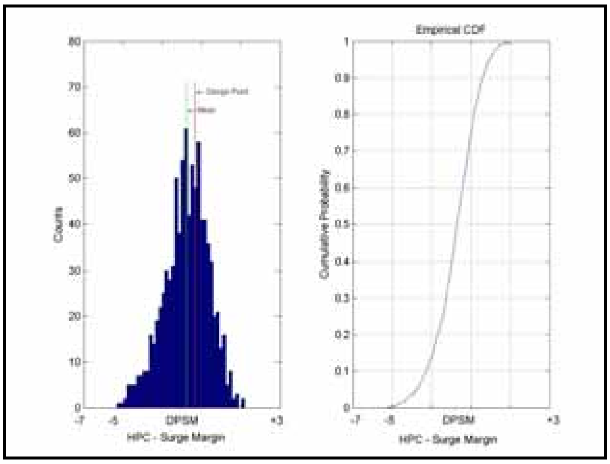


FIGURE 5. HPC surge margin.

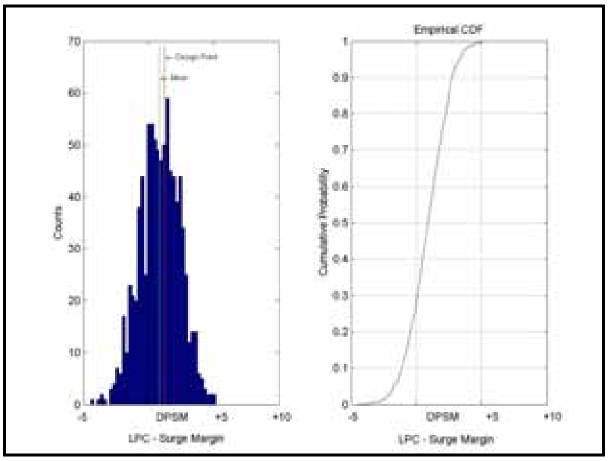


FIGURE 6. LPC surge margin.

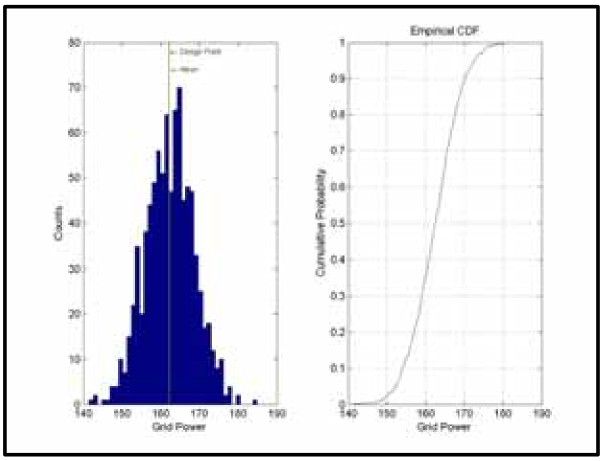


FIGURE 7. Grid power.

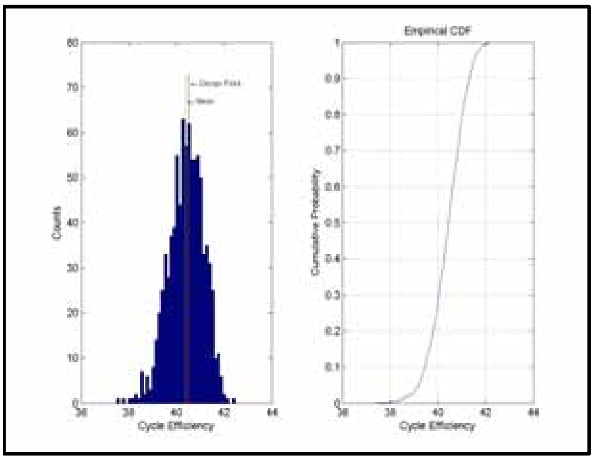


FIGURE 8. Cycle efficiency.

From the distributions shown in FIGURE 3 to FIGURE 8, it can be seen that the mean value of the distributions corresponds closely to the design point values as would be expected. These distributions also fall within the expected design envelope of the turbo machines and the plant as a whole. The Monte Carlo study was also employed to evaluate the other three steady states and for each case, the results proved to be within the desired envelope.

6. CONCLUSION

From the study, the following conclusions are evident:

- The most sensitive parameters influencing the turbo machines, grid power and cycle efficiency are identified.
- Wide variations used in the Monte Carlo runs produced values that fall within the acceptable rotational speed range.
- The grid power variation on the full power, full inventory steady state produced a variation that was within the generator specification of 180 MWe.

- The cycle efficiency variation is within the assumptions made for the PBMR business case.

Because of the study, the design effort can be focused on the most sensitive parameters and the plant and its major components are optimized more easily. The results are, however, only as good as the modeling and the assumptions made, and thus time will tell if the nail was hit on the head. At this stage the confidence level is high that no important parameter has slipped through unnoticed.

The robustness of the design is demonstrated by the Monte Carlo result distributions. In all steady state cases that encompass the entire operating envelope, the output variation fell within the turbo machine and plant design envelope.

This study only looked at steady state sensitivity and Monte Carlo analysis. Work has also been done on transient Monte Carlo analysis of the reactor during various upset conditions. The thermo hydraulic code is also being upgraded to do the sensitivity and Monte Carlo analysis without the need for any external automating software.

Finally, sensitivity and Monte Carlo analysis is carried out continuously on all systems and subsystems to gain understanding of the system itself, and to gain confidence that the variance in output would fall within the specified design envelope. This capability is becoming an ever increasingly powerful analysis tool in the design of the PBMR MPS and all its subsystems.

ACKNOWLEDGEMENT

It is with great appreciation that everyone at PBMR who provided input, insight and advice to this study, is acknowledged for their contribution.

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AUTHOR INTRODUCTION

Erik van der Linde, born in 1975, is presently working as a simulation design engineer in the Thermo-Hydraulic Analysis Group within PBMR. He received a Bachelors degree in mechanical engineering (1997) from Potchefstroom University and is at present enrolled for a Masters degree at the same university. He was employed at a steel mill from 1998 to 2001. During this time he was involved in diverse projects, from upgrading blast furnace cooling water flow to building a new compressed air plant. His employment with PBMR commenced in 2002 in the turbo machinery section.