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Idealized 100-year Nuclear Growth Scenarios in One or Two-Region World Models of Slow-Growing Industrialized and Fast-Growing Developing Nations

***IAEA: 2nd Research Coordination Meeting of a CRP on
Small Reactors Without Onsite Refueling***

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***David C. Wade
Argonne National Laboratory***



U.S. Department
of Energy

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Outline

Background

- A Proposed Nuclear Architecture to Fuel Sustainable Development
- Idealized Modeling of a Transitioning Nuclear Energy Park

Phase 1: One-Region World Model: Is there sufficient ore?

- Market Penetration strategies (Mix of reactor Types) to Merely Maintain Market Share
- Strategies to Dramatically Increase Market Share

Phase 2: A Two Region World: What are the benefits of trade?

- Status of Modeling

Summary

- Strategies for Optimal Allocation of Fissile Resources during a 100 year Transition to Nuclear-Fueled Sustainable Development

Background

- Most of the recent scenario studies of nuclear futures
 - Focus on maintaining market share
 - Often are confined to
 - *Waste Management for Developed Countries*
- Thus – nuclear growth rates are modest in these studies

At the same time

- GIF and INPRO place “sustainable development” goals on nuclear development

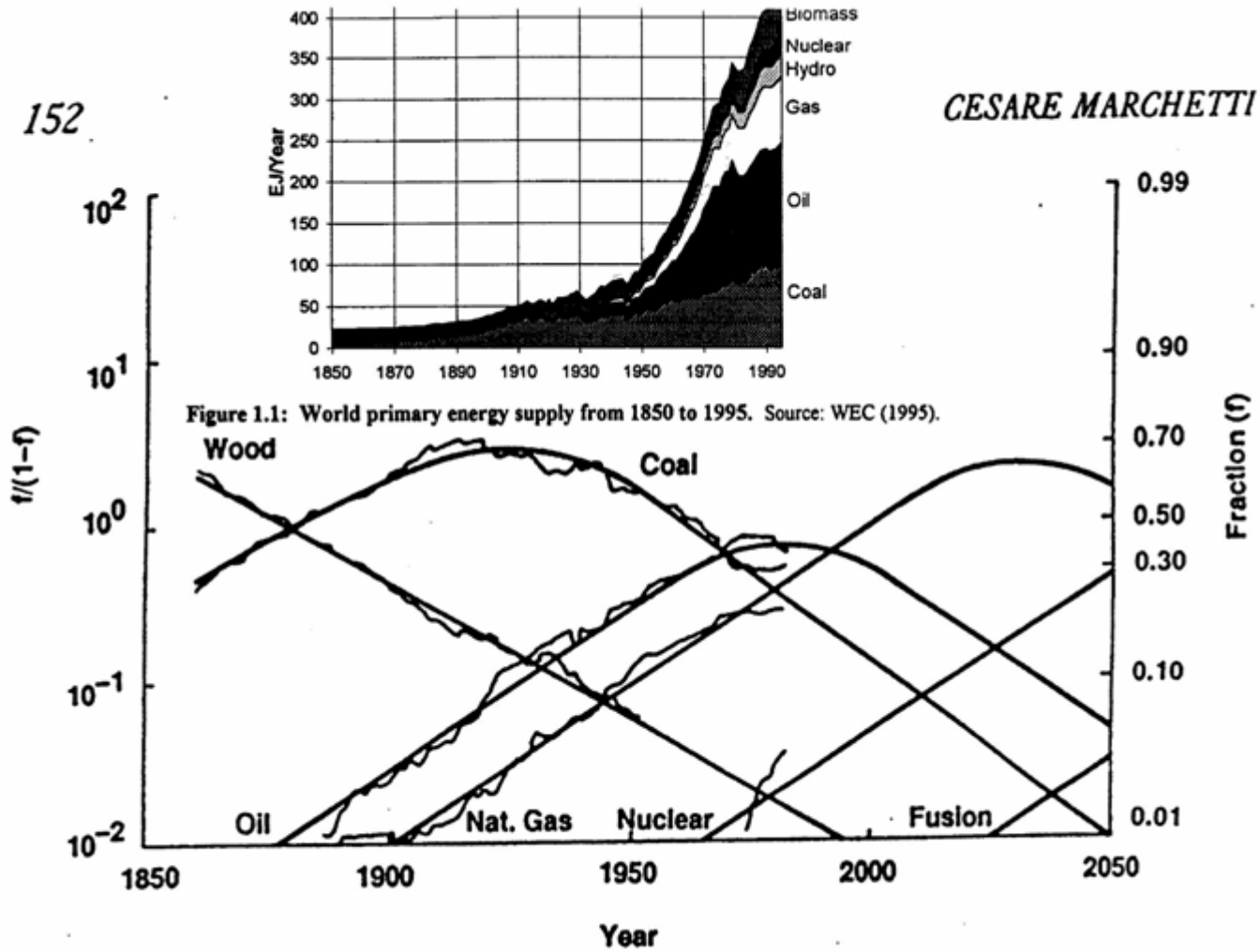
So

- Here we will analyze century-long market penetration strategies for
 - Significant nuclear growth in a Hub-Spoke Architecture
 - *Expanded service areas (developing countries)*
 - *Expanded energy products (Water, H₂, process heat)*

Attaining

- Major market share of world primary energy by Century’s end
- Constrained by fissile availability and dynamic response properties of nuclear systems

Historical Market Penetrations To Increase Market Share



Market Share of World Primary Energy

Goals for The Study

- Using an Idealized Model:
 - Optimize a constrained 100 to 150 year dynamic transition
 - From the current LWR Once-Thru Cycleto
 - A fissile-self-sufficient, growing, global nuclear energy park
 - *Comprised of:*
 - + LWR's ($CR \ll 1$)
 - + STARs ($CR \approx 1$) representative of small fast reactors of long refueling interval
 - + FBR's ($CR \gg 1$) fast breeder reactors
 - Which attains significant market share of world primary energy
- Find the time-dependent optimal allocations of fissile material and time evolving mix of
 - Reactor types: LWR, STAR, FBR
- Constrained by both:
 - A specified nuclear park growth rate, $e^{\alpha_N t}$and
 - An upper bound on available virgin U235 from ore reserves
- Employ Optimization Methods from Classical Mechanics

A Two Year Program of Study

■ Year 1 (completed)

- A one region world with specified nuclear growth rates
- Constrained by U ore

Find out vs growth rate

- What evolving mix of fueling strategies and mix of reactor types can transition to a sustainable (fissile-self generating) world nuclear park

■ Year 2 (in progress)

- A two-region world
 - *Developing countries growing fast from zero initial deployment*
 - *Developed countries growing slowly from a nonzero initial deployment and a large inventory of legacy spent fuel*
- What (different) evolving mixes of reactor types in regional energy parks can transition to a sustainable nuclear park
 - *With trade of fissile mass between regions restricted*

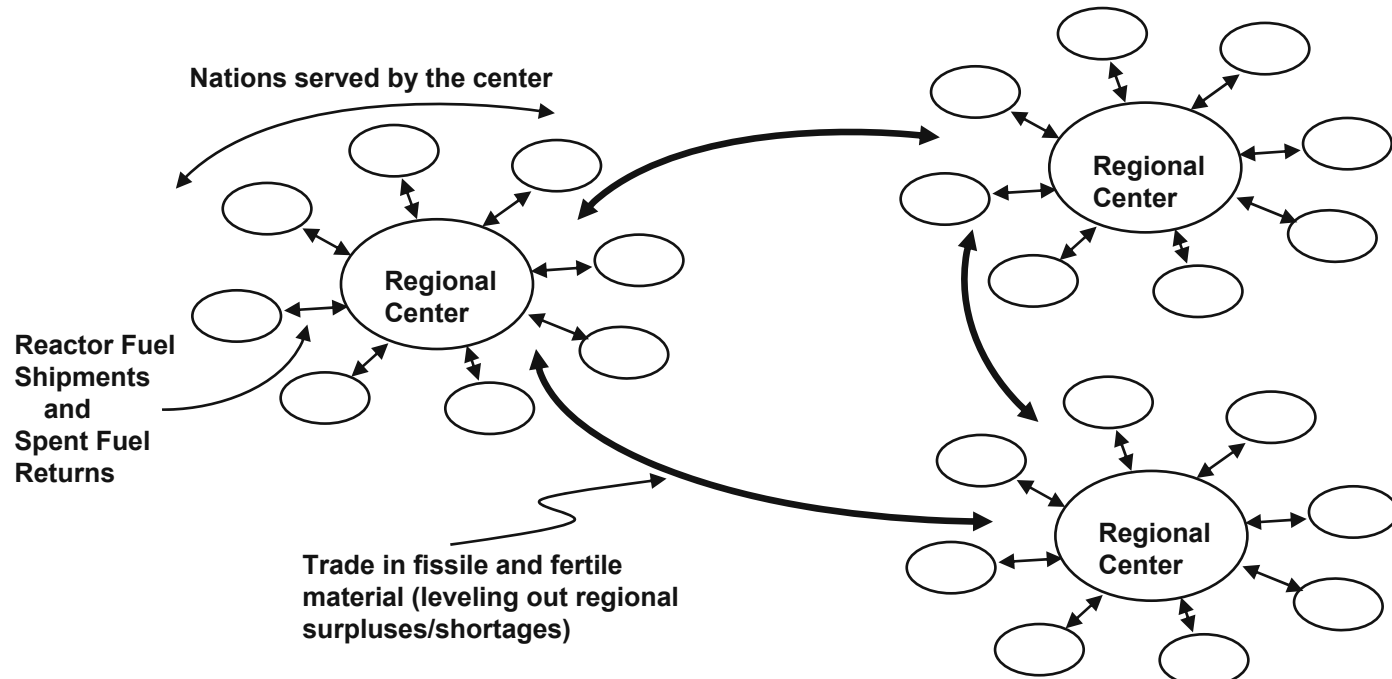
vs

- *With trade allowed*

- – But no outcomes to show yet – sorry!)

The Proposed World Nuclear Energy Park

- Centralized Regional Fuel Cycle Centers
 - Facilities for: enrichment, recycle, fabrication, waste management
 - Co-sited Breeder Reactors – (Economy of Scale Sized)
- Distributed Power Plants at Customer Sites
 - LWR's, HTGR's, etc., (thermal spectrum)
 - STARs long refueling interval (fast reactors) of small to medium power rating



- Breeders are sited only at the Regional Fuel Cycle Centers

Why Regional Fuel Cycle Centers

- The energy density of nuclear fuel
 - Exceeds by 10^6 that of chemical fuel
 - Making it a preferable long distance energy carrier
 - 6 to 12 Regional Centers could deliver the World's entire primary energy usage throughout this century
 - These could service those countries who
 - Don't want the expense of an indigenous infrastructureand would
 - Concentrate the safeguards activities where handling of bulk fissile takes place
- and
- Benefit from Economy of Scale of the Fuel Cycle Facilities

Two approaches are under Discussion:

- Multinational Regional Centers (IAEA/Russian Approach)
- Supplier States/User States (US - GNEP Approach)
- Both display the attributes enumerated above

Why Distributed LWR's?

- LWR's are proven and available in the near term
 - Developed countries “like” them
- Some fast-growing developing countries (China, India)
 - Already have significant grid emplacement

so:

- Can Accommodate Economy of Scale Sized Power Plants

and:

- They plan to be Totally Self Reliant (Emplace Indigenous Fuel Cycle Infrastructure) – so frequent refueling does not exacerbate rates of international fuel shipments

- LWR's will be deployed in significant numbers
 - Now to ~2040-2050
- They have multi-decade lifetimes – i.e., once built, they will last the whole century
- They are an inevitable component of the evolving mix
 - Their initial ore requirement/GW_t deployed is small

but:

- Their lifetime consumption of ore is considerable

Why Distributed STAR's?

- STARs are designed for the Developing Country Market
 - “Right-sized” for initially small but fast growing grids
 - Energy security (20y refueling) for countries who (given guarantees of service from a regional center) don't want the expense of an indigenous fuel cycle and waste repository infrastructure
 - Shipping of fuel is done infrequently in big (20 year refueling cassette) discrete batches
 - Whole core cassette refueling every 2 decades; item accountancy
 - Fuel entombed in frozen Pb during shipping
 - STAR's require a very large initial fissile working inventory but
 - Their one time initial loading of fissile is actually less than an LWR's lifetime consumption of U235 – for the same energy delivery
- and
- Once loaded, they are fissile self sufficient i.e., $CR \cong 1.0$

Why Economy of Scale Breeders Sited At Regional Centers?

- Powerful Breeders (high BR & Short Doubling Time) are required
 - To manufacture fissile fuel when ore is exhausted
- Doubling time is shortened when
 - BR increases, (big reactors \implies neutron economy) and when
 - Specific power (kw/kg fissile) and power density (kw/l) are high, \implies Na-cooled fast reactors at high kw/l
 - Out of reactor recycle time is short \implies so co-site them with recycle plants
- Their function is to create long distance energy carriers (nuclear fuel) at the regional fuel cycle centers*
 - Manufacture fissile fuel energy carrier – from U238 (& Th 232)
 - Use released heat to manufacture H₂ based synthetic chemical fuel energy carrier
 - *From Water at $\leq 50\%$ efficiency (LHV H₂ heat/nuclear heat)*
 - i.e., they don't make electricity (which is a relatively short distance energy carrier)

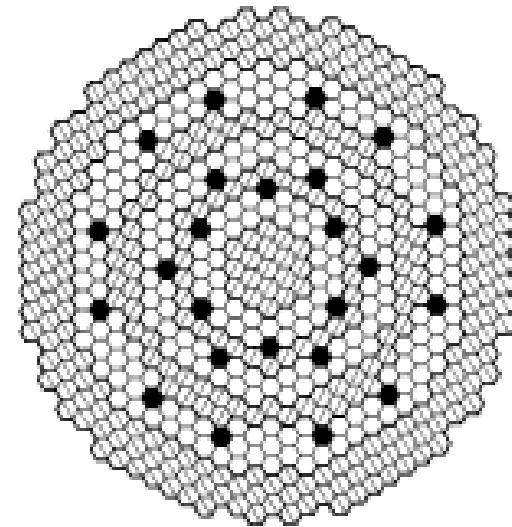
* Analogous to centralized oil fields of 20th Century; coal mines of 19th century

High Breeding Gain Na Breeder

- The Na Breeder of BR= 1.72 is a 1000 MWe, metal fueled, Na cooled FBR of radially heterogeneous layout
- Designed at Argonne as part of the INFCE activity in 1979
- Documented in Table XXX111, pp 176-179, Vol. 5 – Fast Breeders, International Nuclear Fuel Cycle Evaluation, Published by IAEA, Vienna (1980)

Note:

- Independent nuclear futures scenarios recently conducted in Russia and in India confirm the necessity of breeders of $BR \geq 1.6$
- China and India have both indicated their plans to use metal-fueling of Na FBR's to meet fissile demands in fast-growing deployment regimes



R. B. Grover and Subhash Chandra, "Scenario for Growth of Electricity in India", Energy Policy, p. 2834-2847, Vol. 34, Issue 17, 2006.

A. Yu Gagarinsky, V. V. Ignat'ev, N. N. Ponomarev-Stepnoi, S. A. Subbotin, and V. F. Tsibul'skii, "Role of Nuclear Power in World Energy Production in the 21st Century", Atomic Energy, Vol. 99, No. 5, 2005.

An Idealized Model for A Dynamic Transition of the World Nuclear Park

$$\frac{d}{dt} \begin{Bmatrix} P_i(t) \\ P_f(t) \\ ore(t) \\ B(t) \\ escrow(t) \end{Bmatrix} = \begin{bmatrix} & & & & \\ & & & & \\ & & & & \\ & & & & \\ & & & & \end{bmatrix} \begin{Bmatrix} P_i(t) \\ P_f(t) \\ ore(t) \\ B(t) \\ escrow(t) \end{Bmatrix} + \begin{bmatrix} \\ \\ \\ B \\ \end{bmatrix} u(t) \quad 0 \leq t \leq 100 \text{ years}$$

- Here

$P_i(t)$ = GW_t from LWR's

$P_f(t)$ = GW_t from STAR's + FBR's; $P_f = f \cdot P_{STAR} + (1-f) P_{FBR}$; user-supplied ratio, f

$B(t)$ = Inventory of TRU from reprocessing LWR and FR fuel

escrow (t) = Escrow of ore set aside for future fueling each time a LWR is deployed

- Here

$u(t)$ = Ore withdrawal rate (k-tonnes of U/year)

[B] = Allocation Matrix to reactor type – (all zeros except row 1)

- Initial Conditions

$$\begin{Bmatrix} P_i(0) \\ P_f(0) \\ ore(0) \\ B(0) \\ escrow(0) \end{Bmatrix} = \begin{bmatrix} 1 \text{ TW}_t \\ 0 \\ 15 \cdot 10^3 \text{ kilotonnes} \\ 0 \\ 0 \end{bmatrix}$$

Note: We use TW_t because it maps directly onto fission rate

$$\frac{1 \text{ gm fissioned}}{MW_t \text{ day}} \Rightarrow \frac{1 \text{ tonne fissioned}}{TW_t \text{ day}}$$

How The Model Works

- The global nuclear energy park is envisioned to contain
 - Large power rating LWR's of 1 yr refueling interval sited at distributed customer sites
 - Small power rating STAR's of 20y refueling interval sited at distributed customer sites
- The FR park energy fractions are prespecified by the analyst: -- input value, f
 - $0 \leq f \leq 1$ from STAR's – characterized by
 - $CR \equiv 1$
 - *Closed fuel cycle with very long refueling interval*
 - *Very large initial working inventory*
 - $(1-f)$ from FBR's – characterized by
 - *Very high Breeding ratio = 1.72*
 - *Standard closed fuel cycle (1 year refueling interval and cycle holdup time = 5y)*
- As the nuclear park grows to meet demand
 - Ore is withdrawn to fuel existing LWR's and build new LWR's
 - *When a new LWR is built, its lifetime ore requirement is placed in escrow*
 - *The LWR's are decommissioned with an average lifetime of LWR life (=35y)*
 - *The LWR's discharge fuel (at 50 MW_t days/kg IHM) contains TRU*
 - The LWR spent fuel is reprocessed to recover the TRU
 - *After an average holdup time in reprocessing/refab of holdup (=5y)*
 - *The TRU is used to fuel initial inventories of a growing FR park*
- The FR's are built as soon as TRU is available to meet power growth demand, but if insufficient,
 - More ore is withdrawn to make up the difference with LWR's – while minimizing PI

Define a Performance Index to Be Optimized by Timing of Ore Withdrawals During A 100-150 Year Transition in The Nuclear Park Configuration

$$PI = \int_0^{100y} \left\{ \left(\frac{P_{park}(t) - P_o(t)}{P_o(t)} \right)^2 * P_o^n(t) \right\} e^{-\alpha_{ige}t} dt \quad \text{where } \begin{cases} n=0; \text{ minimize \% deviation squared} \\ n=2; \text{ minimize absolute dev. squared} \end{cases}$$

Here:

- $P_{park}(t) = P_{LWR}(t) + P_{STAR}(t) + P_{Breeder}(t)$
- $P_{world}(t) = 16 \text{ TW}_t e^{\alpha t}$ = growth of world primary energy demand
- $P_o(t) = 1 \text{ TW}_t e^{\alpha_N t}$ = target growth rate of nuclear power
 - $\left(\frac{\alpha_N}{\alpha} \right) \begin{cases} = 1 \text{ means } \underline{\text{maintain}} \text{ current market share (6\%)} \text{ of primary energy} \\ < 1 \text{ means nuclear } \underline{\text{closeout}} \\ > 1 \text{ means } \underline{\text{increase}} \text{ nuclear market share of primary energy} \end{cases}$
- α_{ige} = intergenerational equity discount factor
- Try to increase deployment of distributed reactors (LWR's + STAR's) vs centralized (Breeder's)

- Will use the Mathematical Methods of Classical Mechanics to minimize the Performance Index by an optimal ore withdrawal schedule vs time

Hamilton's Principle and The Calculus of Variations

- The mathematical methods of classical mechanics are used to:
 - Find the optimal ore withdrawal sequence vs time ($U^*(t)$) k tonnes ore/year
 - Which will minimize the PI
 - Subject to the initial conditions, and reactor park dynamic response
 - Subject to the constraints
 - $0 \leq \text{Ore}(t) \leq 15 \cdot 10^3$ kilotonnes *for all t*
 - $U(t) \geq 0$ *for all t*
- Specifically we:
 - Define the Hamiltonian as:
 - *The integrand of the PI*
 - *Plus the product of Lagrange Multipliers $\lambda_i(t)$ times the systems equations*
 - Employ the calculus of variations to derive the Euler equations and final conditions on the adjoint variables, $\lambda_i(t)$,
 - Choose a tentative control; solve system equations forward in time; solve adjoint eqns. backwards in time – then
 - Iteratively update the candidate control, $U(t)$, knowing the 1st variation of the Hamiltonian wrt control (as a function of the adjoints)
 - We use MATLAB and its Control System Toolbox to solve numerically

Numerical Attributes of The Reactor Park

- The Ore and LWR future fueling Escrow are in (kilo tonnes of U present in ore resources)
 - LWR enrichment = 4.2%
 - Enrichment tails = 0.3%
- The Reactor Powers are in (GW_t)
- The Reactors are characterized by Initial Incore Working Inventory (tonnes fissile/ GW_t) and Transuranic Mass (tonnes) produced and fissioned per GW_t year

	Initial Fissile Incore Inventory (tonnes fissile/ GW_t)	TRU Produced per GW_t y (tonnes fissile)	Fissile Fissioned per GW_t y (tonnes fissile)	Discharge Burnup $\text{MW}_t\text{d}/\text{kg HM}$
LWR	1.398	0.09271	0.34675	50
STAR	8.910	0.34675	0.34675	100
FBR	3.310	0.59641	0.34675	100

- The LWRs have an average lifetime of
 - LWR lifetime = 35y; every year $(1/35) \cdot P_t(t)$ GW_t of LWR's are decommissioned
 - When an LWR is built, 35y's of its annual ore requirement is put into escrow
- The STAR and FBR fractions are f and $(1-f)$ respectively of the FR component of the park
- The TRU in the Recycle Buffer stays there for an average life of
 - Holdup = 5y; every year $(1/5) \cdot B(t)$ tonnes of TRU are released
 - Upon release it is used to build STAR's and FBR (in the user specified ratio, f)

Strategy To Maintain Market Share

What Would It Take to Merely Maintain Market Share?

- Suppose by 100 years from now

- 10 billion people, all consuming primary energy

- At current primary energy use of Europe ; $4 \frac{toe}{capita\ year} \left(: 5.3 \frac{kw_t\ year}{capita\ year} \right)$

$$P_{world} = 10 \times 10^9 * 5.3 * 10^{-9} \frac{TW_t}{kw_t} (TW_t\ y / y)$$

$$P_{world} (100) = 53\ TW_t\ y / y$$

- Starting from current world primary energy = 16 $TW_t\ y/y$
world primary growth rate

$$= \alpha = \frac{1}{100} \ln \left\{ \frac{53}{16} \right\} = 0.012$$

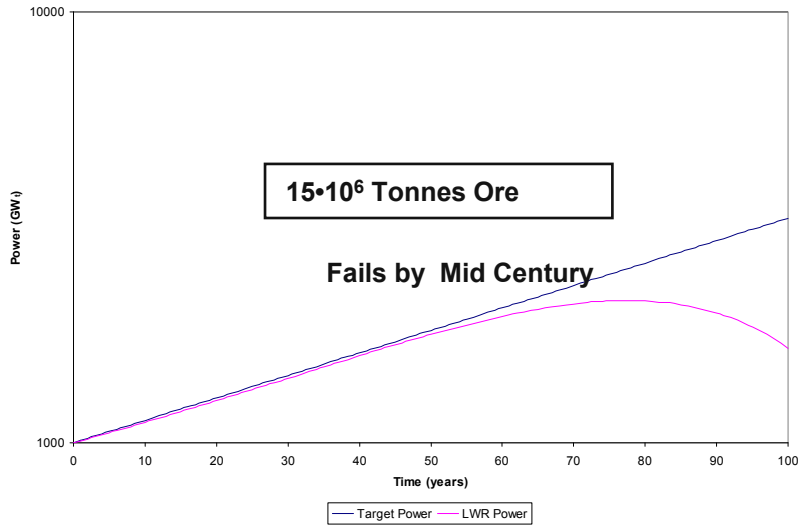
$$\alpha_{world} = 0.012$$

Note: A growth rate of 1.2%/year is at the low end of numerous estimates:

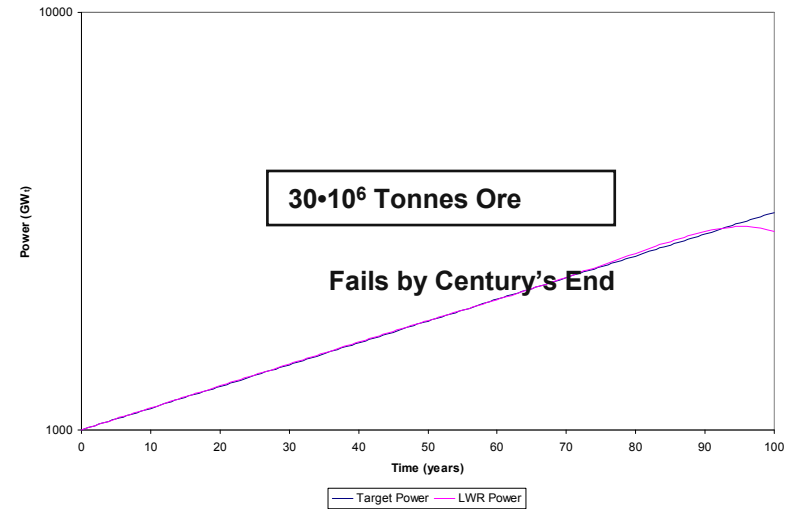
- IEA (2006 est) 2.5%/y from 2002 to 2030
- WEC/IIASA (1998) .8 to 2.2%/y from '99 to 2050
- .7 to 1.6 from '99 to 2100
- DOE-EIA (2006) 2.0 to 2030

Case 1: Maintain Market Share (6%); Strategy: Once Thru LWR; Result: Fails by Mid Century

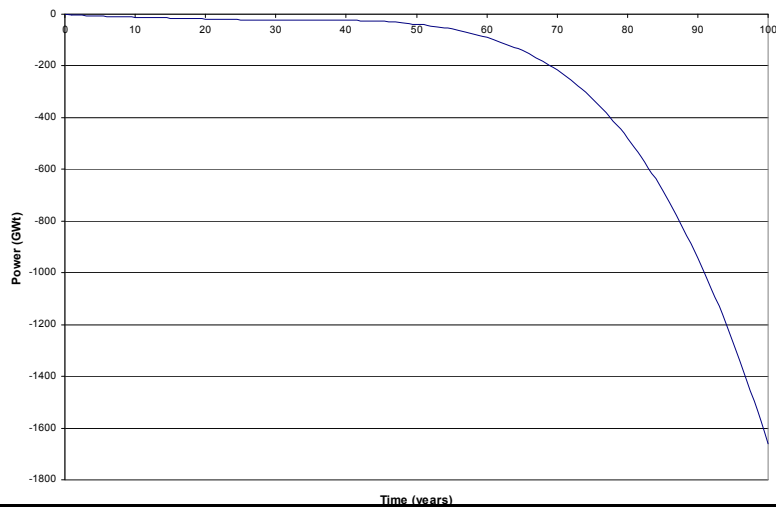
Once Through Power



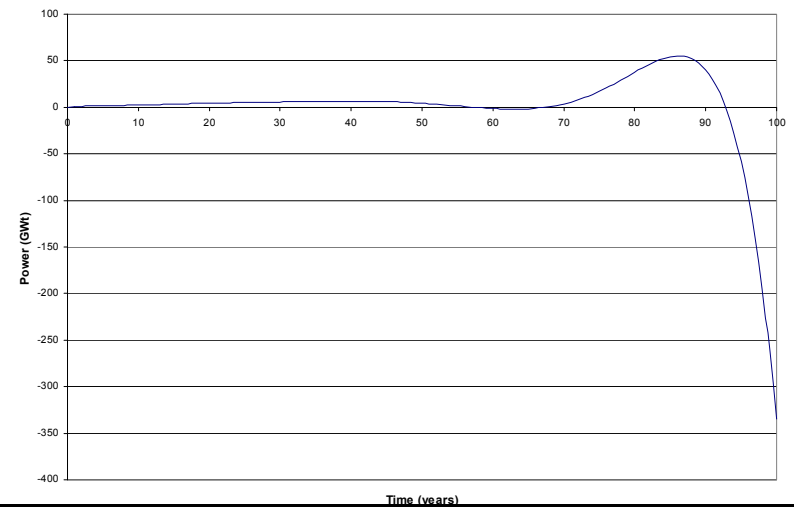
Once Through Power



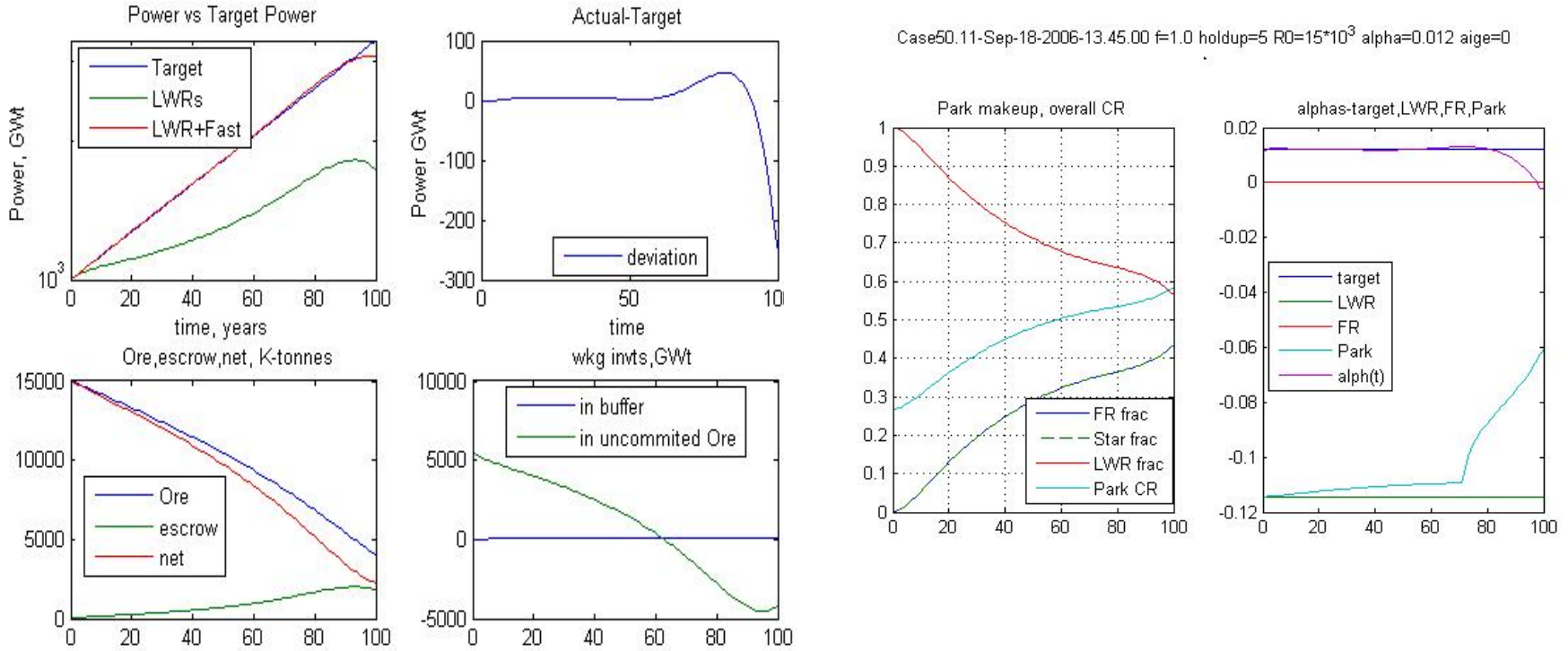
Deviation



Deviation



Case 2: Maintain Market Share (6%); Ore = $15 \cdot 10^6$ Tonnes Strategy: Once Thru LWR \rightarrow TRU Fueling of STAR's (CR=1)



■ Fueling STAR's with TRU from LWR will take us to 2100 on $15 \cdot 10^6$ tonnes ore

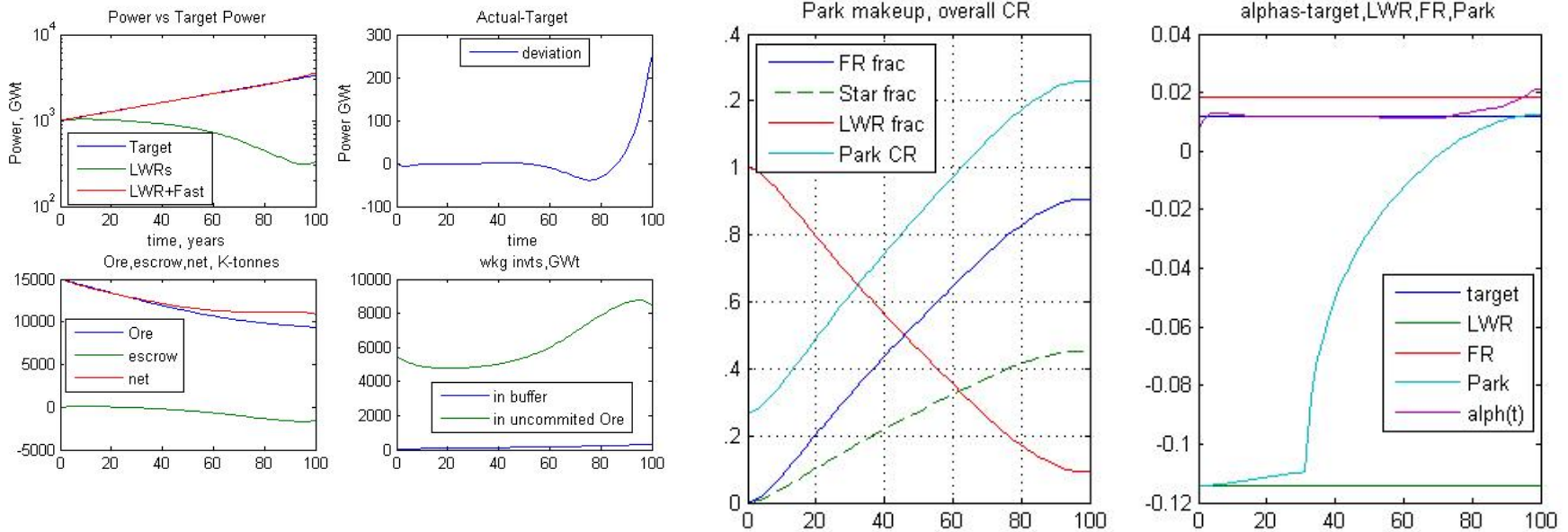
But

■ Park is far from fissile self sufficient when ore is getting low

Case 3: Maintain Market Share (6%); Ore = $15 \cdot 10^6$ tonnes

Strategy: Once Thru LWR \rightarrow TRU Fueling of STAR's & FBR's

Case50.4-Sep-18-2006-14.13.05 f=0.5 holdup=5 R0= $15 \cdot 10^3$ alpha=0.012 aige=0



- Transition is achievable with 50% STAR/10% LWR/40% Breeder by 2100
- Park is Fissile Self Sustaining by End of the Transition
- Substantial Ore Remains
- Distributed Reactors (LWR + STARs) are > 50% Nuclear Market Share

Strategies To Aggressively Increase Market Share

A Significant Contribution to Sustainable Development – What Would it Take?

- Continue to Assume 1.2%/y growth of primary energy
 - 53 TW_t y/y by 2100 – up from 16 TW_t y/y today)
- Suppose by 2100 fossil fuel consumption is to be capped at a value no larger than its current value

$$P_{\text{fossil}}(100) = 0.9 * 16 = 14.4$$

$$P_{\text{nuclear}}(100) = 53 - 14.4 = 38.8$$

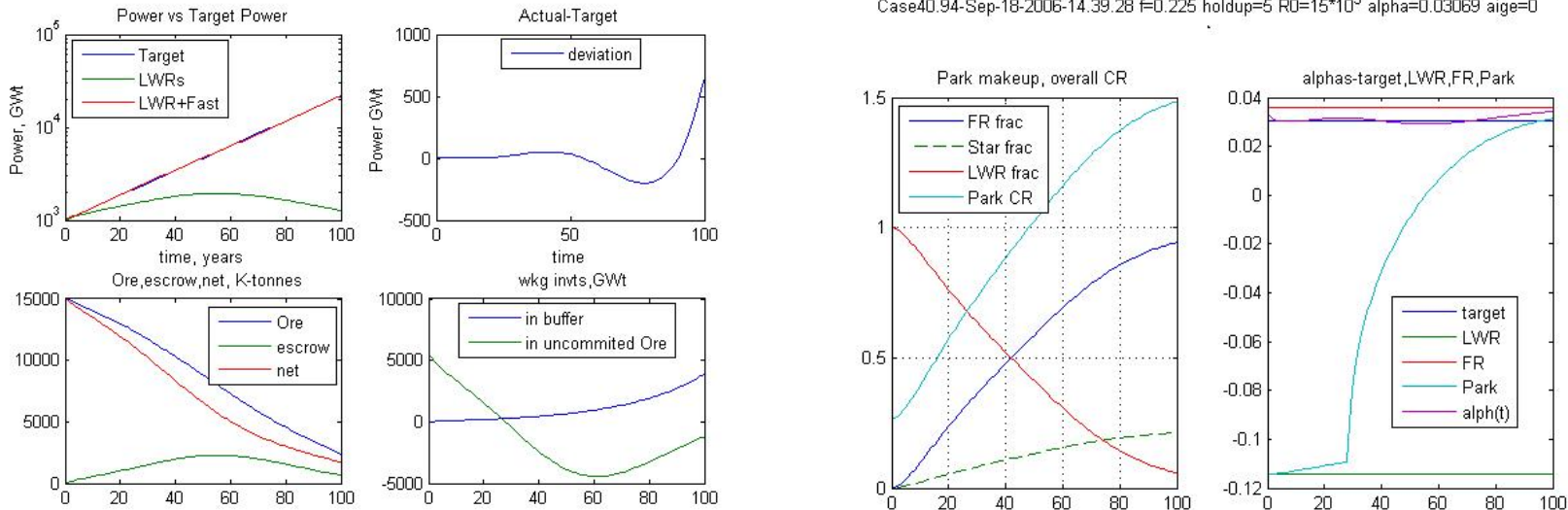
$P_{\text{nuclear}}(100) = 73.2\%$ of world delivered primary energy in order to maintain CO₂ emissions to today's emissions levels

- This is a factor of 39 increase in nuclear over 100 years!
 - Nuclear would have to grow at $\alpha_N = 3.66$ %/year
- Is it technically feasible in a world constrained by ore and dynamic response?

Case 4: Maintain Fossil Use at Current Level Increase Nuclear Market Share to 75% by 2100

Strategy: LWR → TRU fueling of STARs and Breeders

Case40.94-Sep-18-2006-14.39.28 f=0.225 holdup=5 R0=15*10³ alpha=0.03069 aige=0



- Transition is achievable with 22% STAR/6% LWR/72% Breeder by 2100
- The nuclear park is fissile self sufficient by the end of the transition
- LWR's double by 2055; then diminish
- Distributed Reactor (LWR + STARs) market share is >50% until mid century and ~25 to 30% by century end

Nuclear Market Penetration of World Primary Energy

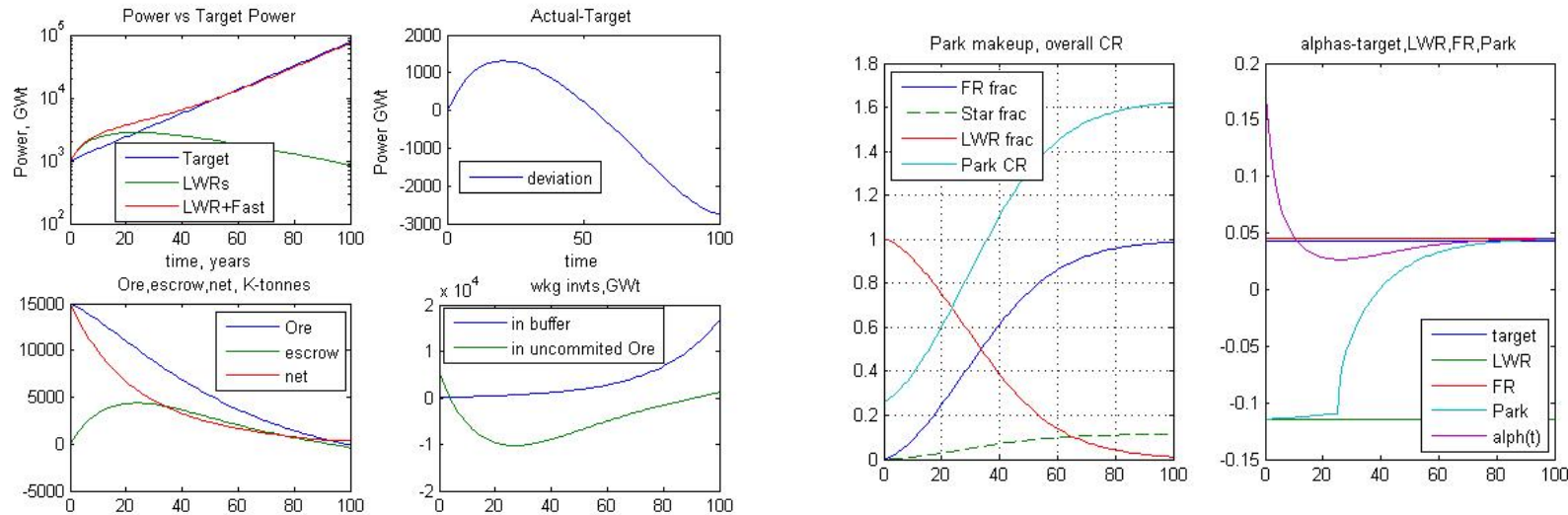
Goal	Nuclear Market Share by 2100 (%)	Nuclear Power by 2100 (TW _t)	100y Growth Rate (%/y)
Maintain Market Share	6	3.18 (~ factor of 3 increase)	1.2
Cap Fossil @ current absolute level	75	39.75 (~ factor of 40 increase)	3.68
Reduce Fossil to ½ current absolute level by manufacture of H ₂ (at η=50%) for 2/3 of primary market	144	76.32 (~ factor of 75 increase)	4.34

•Assumes World Primary Energy grows at 1.2%/y from 16 TW_t to 53 TW_t over a 100y period
(53 TW_t would support 10 Billion people at 4 toe/capita)

†Current nuclear market share is ~6% of total primary energy of 16 TW_t

Case 5: Reduce Fossil to ½ Current Usage by Nuclear Production of Electricity and of H₂ at 50% Efficiency Heat → H₂ LHV

Case60.4-Sep-18-2006-16.17.56 f=0.115 holdup=5 R0=15*10³ alpha=0.0434 aige=0



- The transition is achievable with ~11% STAR/2% LWR/87% Breeder by 2100
- The nuclear park is fissile self sufficient by the end of the transition

■ LWR's are substantially overbuilt for 1st 25 years

But

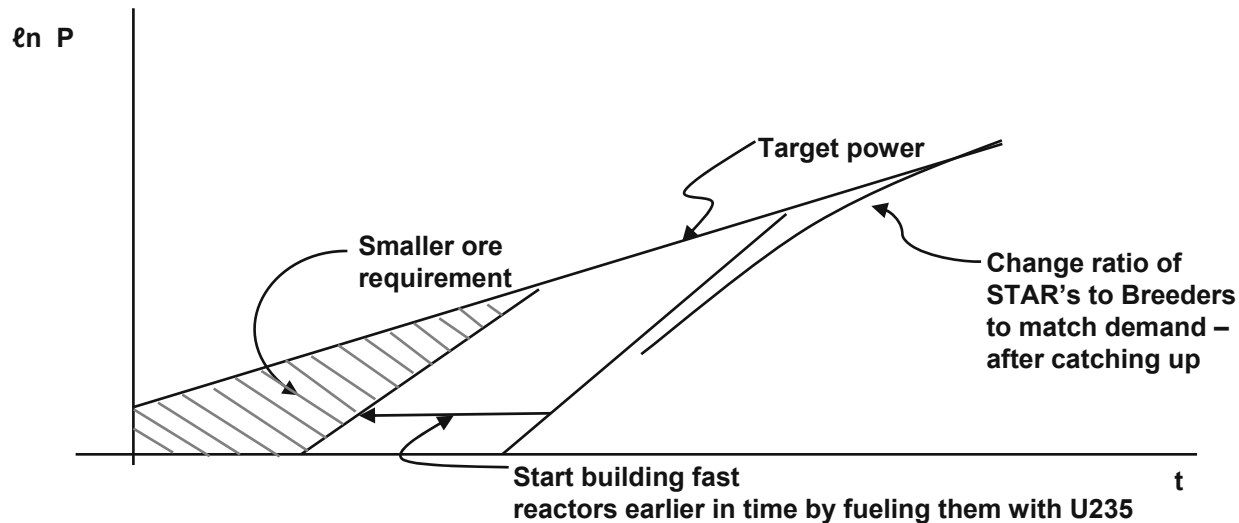
- Distributed Reactors (LWR + STAR's) are down to <15% Nuclear Market Share late in the century; the rest are centrally sited Breeders

A Model With More Degrees of Freedom For The Control

- We need to allow
 - STAR vs Breeder to vary with time

and we can't wait for TRU from LWR spent fuel to build fast reactors

- We need to get an earlier start on fast reactor deployment (both Breeders & STARs) – use U235 fueling



- So – We need a six component control vector
 - U1 (t) = ore withdrawal for building and fueling LWR's
 - U2 (t) = ore withdrawal for building STAR's
 - U3 (t) = ore withdrawal for building Breeders
 - U4 (t) = TRU withdrawal for building and fueling LWR's
 - U5 (t) = TRU withdrawal for building STAR's
 - U6 (t) = TRU withdrawal for building Breeders
- } accelerates fast reactor deployments by fueling with virgin U235

Does Use of Virgin U235 to Build Fast Reactors Make Sense?

- The purpose is to achieve an earlier start on Fast Reactor deployments
 - For putting scarce fissile mass into reactors of higher conversion ratio early in the transition
- We pay a penalty in initial working inventory

	Initial Fissile Incore Inventory (tonnes fissile/GW _t)	TRU Produced per GW _t y (tonnes fissile)	Fissile Fissioned per GW _t y (tonnes fissile)	Discharge Burnup MW _t d/kg HM
LWR	1.398	0.09271	0.34675	50
STAR	8.910	0.34675	0.34675	100
FBR	3.310	0.59641	0.34675	100

- But no penalty when account is taken for the lifetime escrow of fissile requirements for LWR's

	Initial	Initial + Escrow Factor	Lifetime Consumption
LWR	1.398	$I_t + K3 \cdot K1 = 12.129$	8.676 (consumption)
STAR	8.910	8.910	0
FBR	3.310	3.310	25.2 (production)*

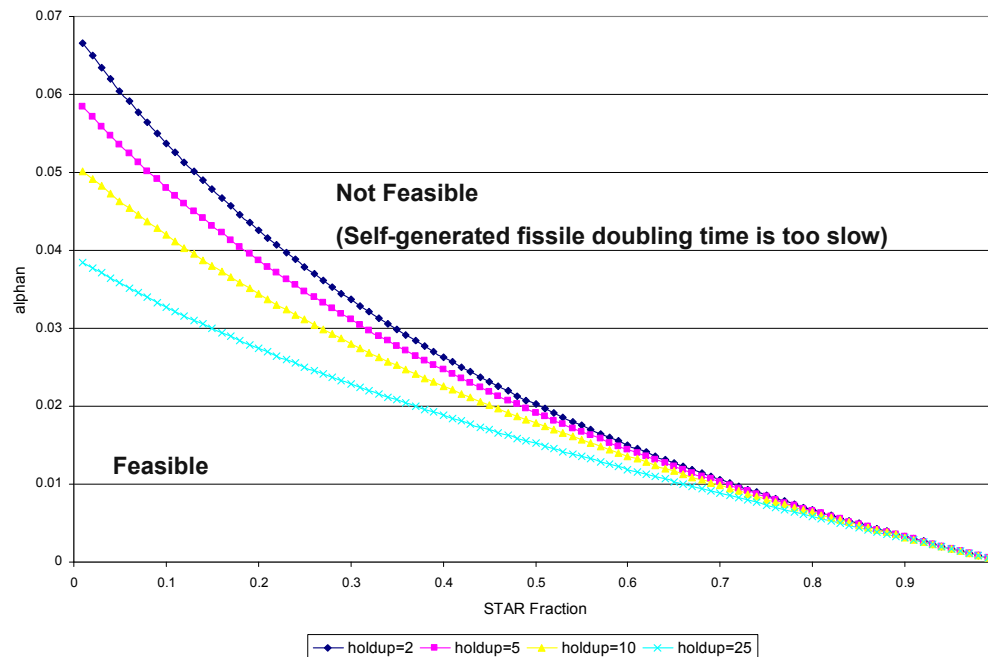
- U235/U enrichment = 15% for STARS' driver fuel 20% for Breeder driver fuel (ore withdrawals per tonne U235 doesn't change much)

$$* \frac{\text{Net fissile production}}{\text{year}} * \text{years of service} = \left(\frac{.59642}{.34675} - 1 \right) 35 = 25.2$$

Dynamic Response

- We want to maximize distributed (LWR's & STAR's) and minimize (centralized) Breeders – BUT
- Limit of Growth Rate for A Fissile-Self-Fueled Fast Reactor Park Comprised of Fraction (f) of STAR's and Fraction ($1-f$) of Breeders
- So – we have to adroitly adjust both the evolving mix of reactor types and the evolving allocation of limited U235

Parametric Survey on Holdup



The Refined System Equations Are

$$\begin{aligned}
 \frac{d}{dt} \begin{bmatrix} P_t \\ P_s \\ P_b \\ \text{recycle} \\ TRU \\ \text{VirginU 235} \\ \text{escrow} \\ w_p \\ P_0 \end{bmatrix} &= \begin{bmatrix} \frac{-1}{LWRlife} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ FP_t & FP_s & FP_b & \frac{-1}{holdup} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -FD & -FD & \frac{1}{holdup} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -K_2 * K_1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_p & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & \alpha_n \end{bmatrix} \begin{bmatrix} P_t \\ P_s \\ P_b \\ \text{repro} \\ TRU \\ \text{VirginU 235} \\ \text{escrow} \\ w_p \\ P_0 \end{bmatrix} + \\
 \begin{bmatrix} \frac{I_t}{I_t + K_6 * LWRlife} & 0 & 0 & \frac{K_1}{I_t} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{I_s u} & 0 & 0 & \frac{1}{I_s} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{I_b u} & 0 & 0 & 0 & \frac{1}{I_b} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & -1 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{K_6 * LWRlife}{I_t + K_6 * LWRlife} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \\ u_5 \\ u_6 \end{Bmatrix}
 \end{aligned}$$

The New Performance Index and Constraints at $t=t_{final}$

$$\text{Minimize } PI = \int_0^{t_f} \left\{ \left[\frac{P(t) - P_o(t)}{P_o(t)} \right]^2 P_o(t)^n (1-W) + W \left(\frac{P_{Breeder}(t)}{P_o(t)} \right)^2 \right\} e^{-\alpha_{ige}t} dt$$

Subject to the System Dynamics Equations, and –

Subject to a fissile availability constraint:

$$(1) \quad + \text{fissile inventory}(t) - \sum u(t) \geq 0 \quad \text{at all values of } t$$

Subject to the Two Constraint Equations at the final time:

$$(2) \quad P(t_f) - P_o(t_f) = 0$$

$$(3) \quad P_{LWR}(t_f) [(FP_{LWR} - FD) - \alpha_T I_t] + P_{STAR}(t_f) [(FP_{STAR} - FD) - \alpha_T I_S] \\ + P_{Breeder}(t_f) [FP_{Breeder} - FD) - \alpha_T I_B] = 0$$

Here

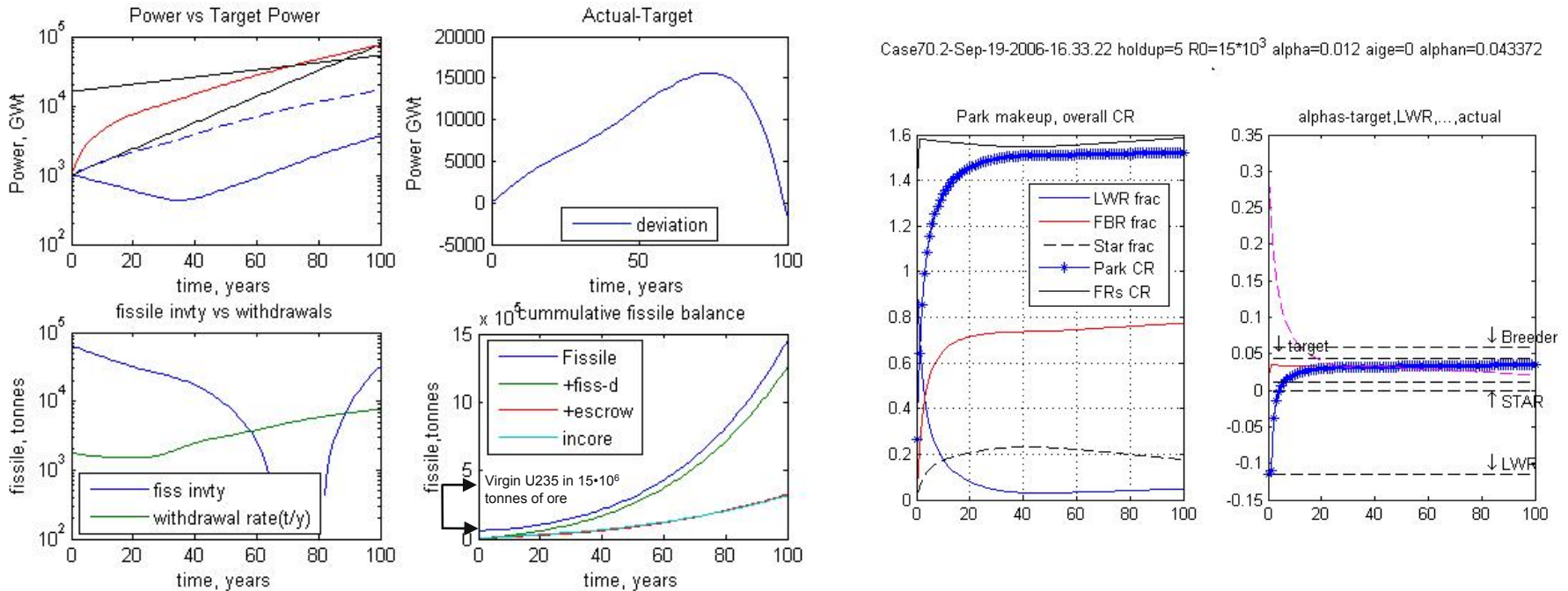
$$n = \begin{cases} 0 \equiv \text{minimize \% deviation squared (balanced near vs long term goal)} \\ 2 \equiv \text{minimize absolute deviation squared} \end{cases} \longrightarrow \text{We use } n = 2$$

$$W = \text{weighting factor on maximizing LWR's + STAR's vs Breeders} \\ \text{(i.e., distributed reactors vs centralized reactors)} \longrightarrow \text{We use } W = 0.3$$

$$\alpha_{ige} = \text{intergenerational equity discount rate} \longrightarrow \text{We use } \alpha_{ige} = 0$$

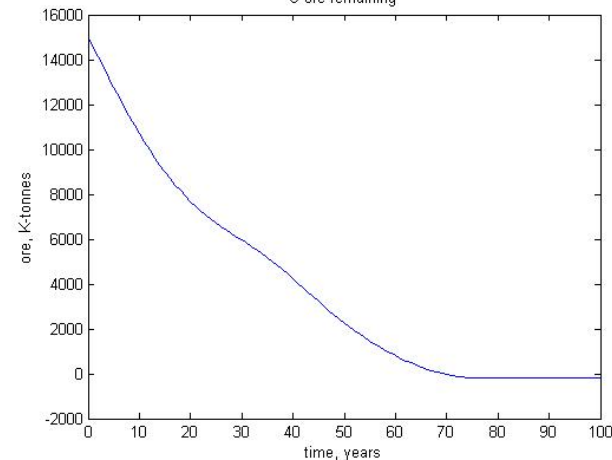
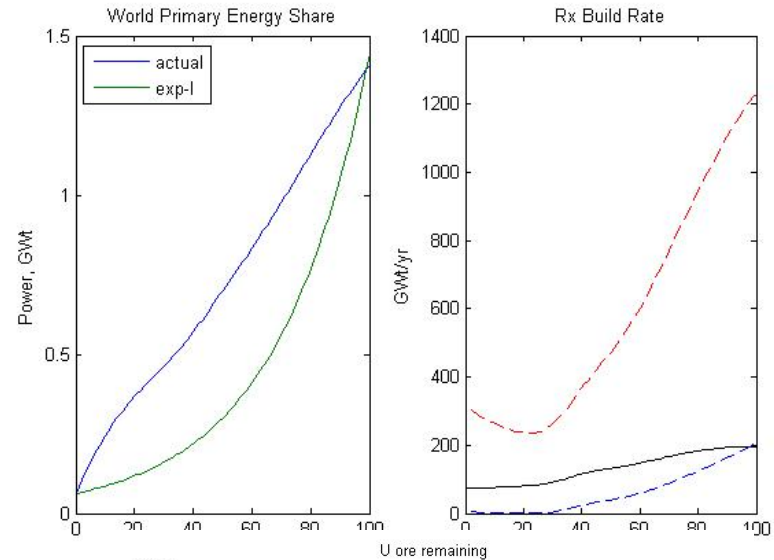
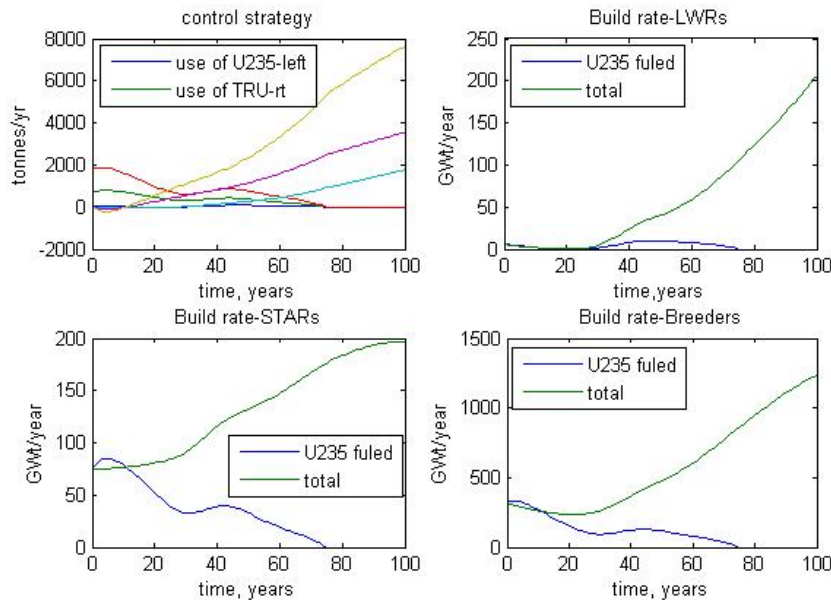
$$\alpha_T = \text{target sustainable development world energy growth rate after } t_f \longrightarrow \text{We use } \alpha_T = 1.2\%/y \\ FP, FD = \text{fissile production/GW}_t \text{ year; fissile consumption/GW}_t \text{ y respectively}$$

Case 7: 100 Year Transition to 144% of Primary Market Share Strategy: U235 Fueling Fast Reactors as Well as LWR's (early) & TRU Fueling of LWR's as Well as Fast Reactors (late)



- Transition is feasible with a park of ~25% STAR's/5% LWR's/70% Breeders
- Fissile self sufficiency is attained very early in the transition
- LWR's decrease until 2040; then increase – fueled by TRU

Case 7: Optimal Allocation Schedule for Virgin U235 and Bred TRU



■ Virgin U235 fuels Breeders and STAR's as well as LWR's until 2070 – when it is exhausted

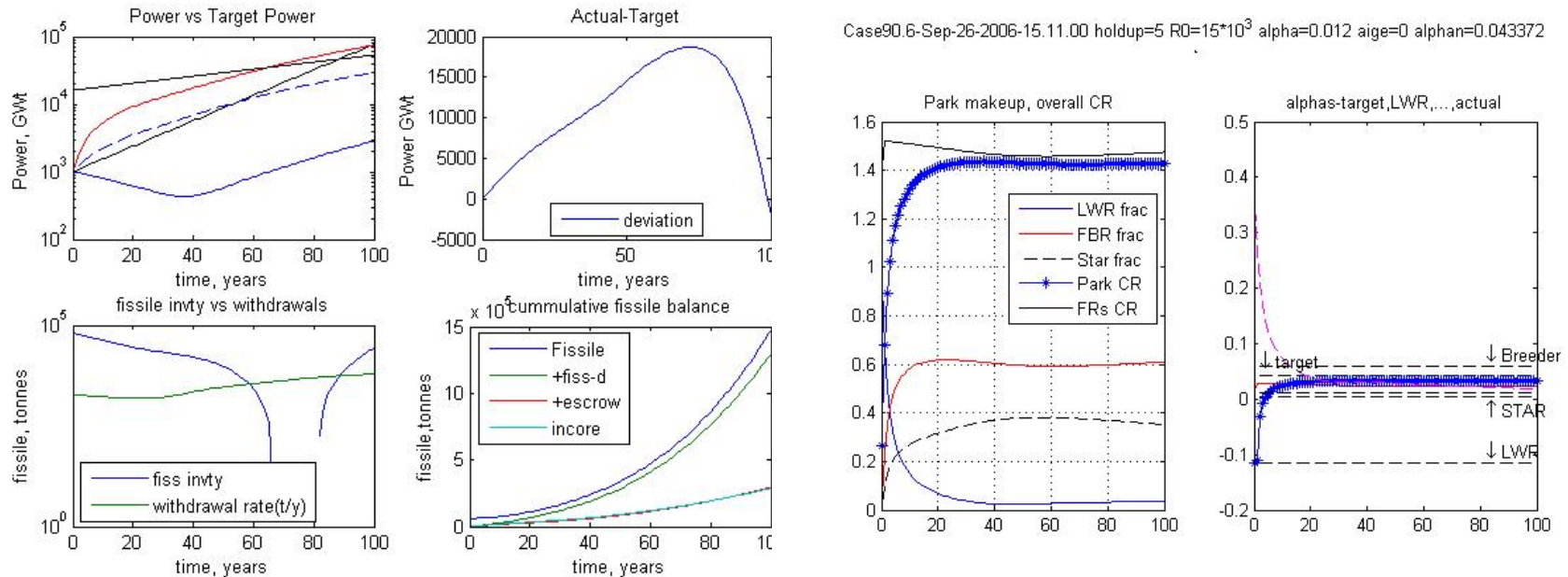
What's Going On? (2)

- This strategy of U235 fueling of Fast Reactors
 - Allows us to raise market share of distributed STAR's up from ~11% to ~20%
- But this still isn't very attractive for interregional equity
 - We'd like distributed reactor/centralized Breeders to be closer to 50/50
- How could STAR market share be increased still further?
 - Parameter Survey with Arbitrary Adjustments of Features*

<u>Case</u>	<u>Effectiveness</u>	<u>Approach</u>
• 150y transition	No	
• Resource = $30 \cdot 10^6$ tonnes	Yes ~30% share	
• LWR with CR=0.75	Marginal ~25%	
• STAR with CR=1.05	Marginal ~25%	Distribute some LWR-SNR pins incore
• STAR with Smaller Inventory	Very ~30%	15y refueling to reduce incore fissile inventory
	~38%	11y refueling to reduce incore fissile inventory
• STAR with 11y Refueling +CR= 1.05	Very ~40%	

* "Arbitrary adjustments" means no new core designs were done – only "estimated"

Case 8: 100 Year Transition to 144% Primary Market Share Strategy: U235 Fueling of FR's & STAR of 11y Refueling & CR=1.05

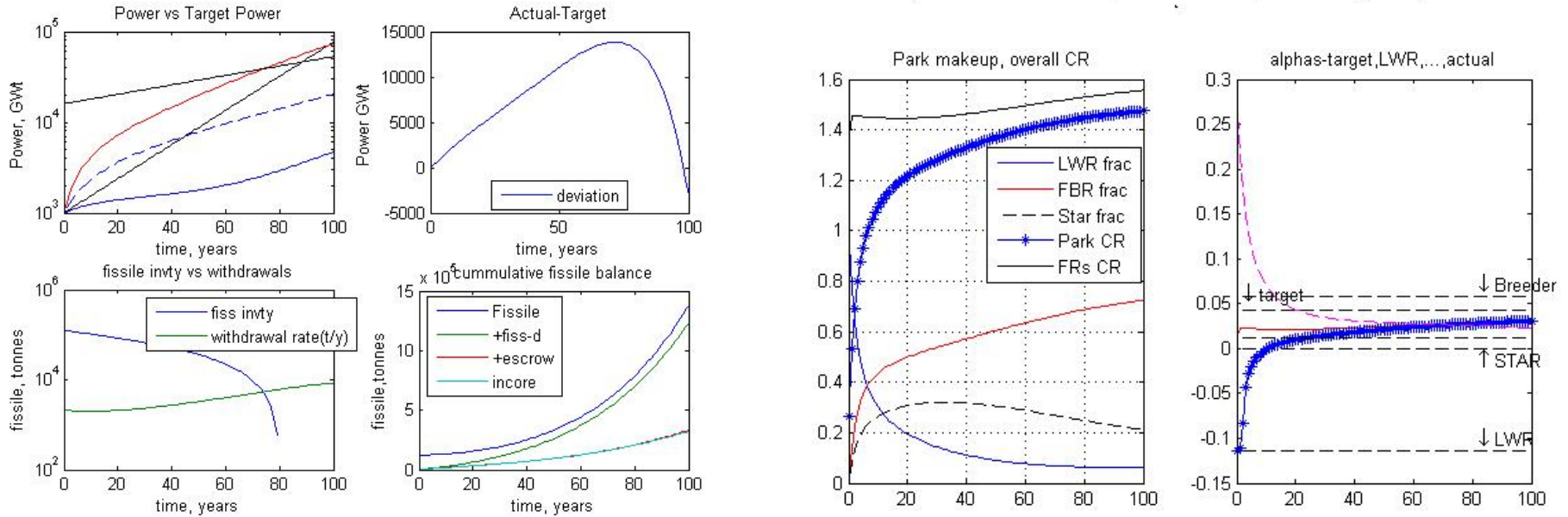


- Cutting back on STAR incore inventory by reducing the refueling interval (20y → 11y)
 - Allows to attain $\geq 40\%$ distributed reactors
- Suggests – Develop high CR thermal reactors (smaller incore inventory)

Case 9: 100 Year Transition to 144% Primary Market Share

Strategy: $30 \cdot 10^6$ Tonnes of Ore

Case90.3-Sep-20-2006-12.53.41 holdup=5 R0=30*10³ alpha=0.012 aige=0 alphan=0.043372



Alternatively:

- Finding more Ore would allow to attain (STAR + LWR) >50% Market Share until 2090

**Summary of Lessons Learned
Using
An Idealized Model
of a One Region World**

Lesson Learned (1)

- For primary energy growth of 1.2%/y to 53 TW_t y/y by 2100
 - Supporting 10 Billion people at 4 toe/capita year (Current use in Europe)
 - A Once-Thru LWR cycle can maintain market share (6% of World Primary Energy)
 - Only until ~2050; Doubling Ore Reserves \Leftrightarrow maintain share to ~2100
 - Introduction of fast reactors (fueled by TRU from LWR SNF) avoids need to find more ore
- and – allows for
- STAR/(STAR + Breeder) ratio up to ~0.6 – achieving both Interregional and Intergenerational Equity at 6% market share

But contribution to Sustainable Development

- Requires much more than maintaining market share at 6% of primary energy

e.g.,

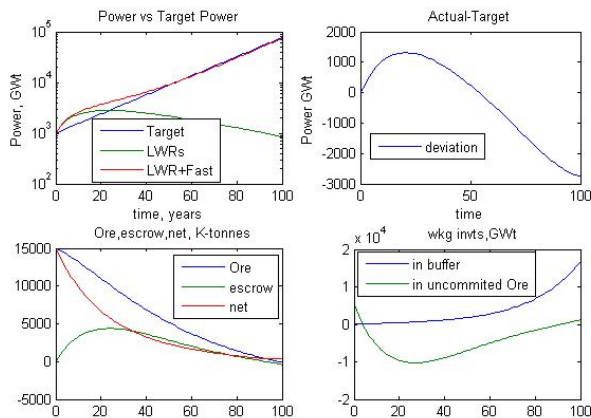
- To hold fossil use at current levels
 - *Nuclear has to grow to 73.2% market share of primary energy by 2100*
- To reduce fossil use to 1/2 of current levels by 2100
 - *Nuclear has to grow to 144% of 53 TW_t y/y primary energy by 2100 (accounting for H_2 manufacture at 50% efficiency for 2/3 of energy delivery)*
- The issue: Is such growth feasible consistent with the laws of physics?

Lessons Learned (2): Even the Most Demanding Growth Would Be Attainable Within Physical Constraints of Ore & Nuclear Dynamic Response

- It is Feasible to Transition to 144% Market Share (76.3 TW_t) by 2100!
 - Within the assumed 15·10⁶ tonnes of U metal present in the ore
- Two Ways To Do It (Identified up to Now)

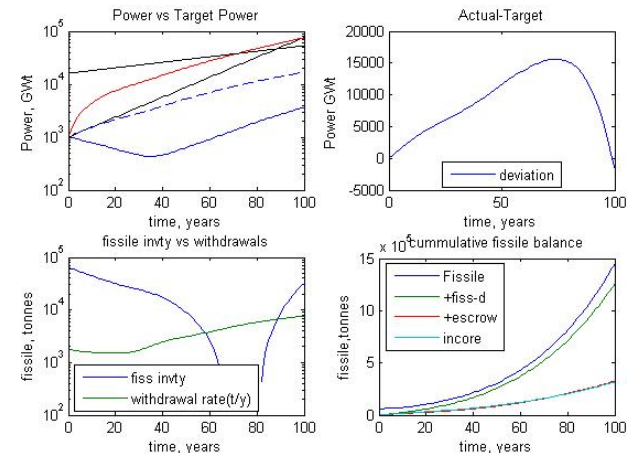
Distributed LWR/Centralized Breeder Strategy

- LWR's fueled by U235
- Breeders fueled by TRU recovered from LWR-SNF
- Breeders dominate the market beyond 2060



Distributed (STAR+LWR)/Centralized Breeder Strategy

- STAR's and Breeders fueled initially by U235 and TRU from LWR-SNF and Breeders
- Centralized Breeders held to 60% market share
- LWRs drop to ~5% market share by 2040



Phase 2: A Two-Region World

Phase 2: A Two Region World Exploit The Complementary Needs of Developing and Developed Countries via Trade

■ Proposed Mutually Beneficial Strategy

(Region A) All Developed Countries

- Slow growth; need to manage SNF
- Reprocess LWR-SNF
 - Set U238 aside
 - “Sequester” TRU into 20 to 30 year STAR refueling cassettes
 - Send only FP to Repository

Complementary Benefits

- Extend repository capacity (as opposed to sending SNF directly to a repository)
- Employ a longer time “sequestration of TRU (as compared to placing TRU in LWR-MOX where spent MOX comes back in 2-3 years)

Simultaneous Benefits

- 21st Century Virgin U235 resource drawdown is reduced (compared to deployment of CR<<1 LWR’s) or dedicated fast burners or ADS’s)
- Less Enrichment is required to be deployed (proliferation hazard reduction)

(Region B) All Developing Countries

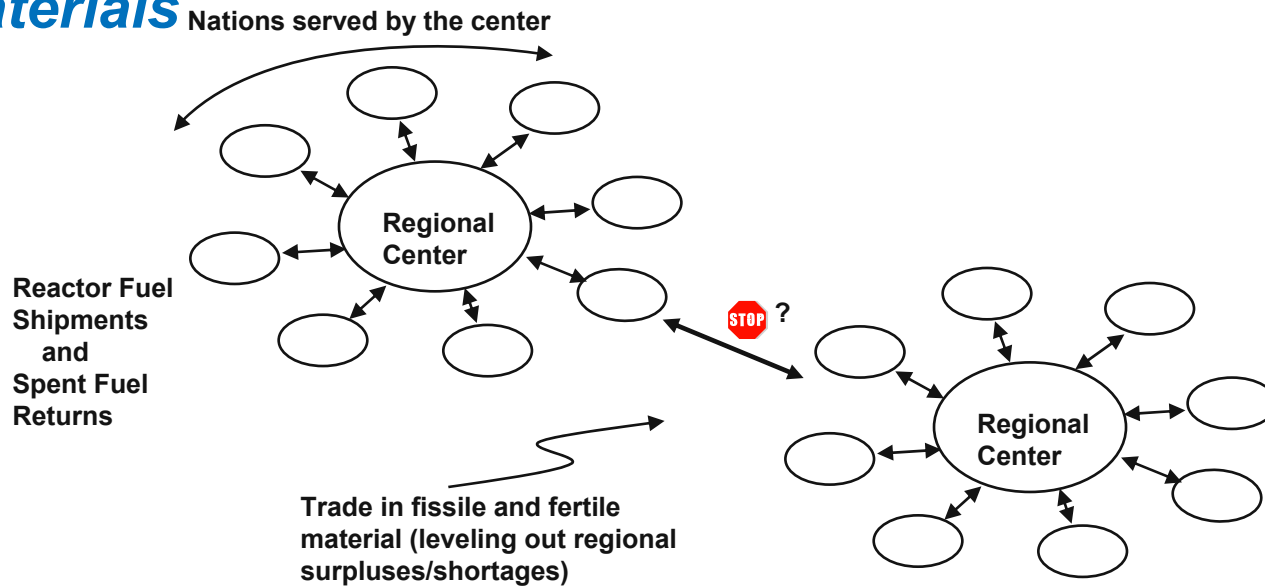
- Fast growth; need to build energy infrastructure

Ship
STAR
Cassettes as
Initial
Working
Inventory

→ Emplace small reactors without onsite refueling with 20-30 year fuel loading

- Deploy reactor systems well suited to the financial need for small initial capital outlay as compared to large LWR deployment combined with indigenous fuel cycle emplacement
- Attain Energy Security by deploying a core load requiring no refueling for 20-30 years

Investigate the Impacts of Trade vs No Trade in Fissile Materials



- (Breeder reactors are sited only at the Regional Fuel Cycle Centers)
- Trade (or not) between Regional Fuel Cycle Centers (not between individual utilities)
- Anticipated Outcomes that would result from Precluding Trade
 - Excess of fissile in some regions/shortages in others
 - *Early in the transition developed regions will be in excess, developing will be short*
 - *Late in transition the situation will reverse*
 - Accelerated drawdown of U ore reserves
 - Buildups of Spent Fuel Inventories

Status of Work on Two Region World Modeling

- Each region has its own
 - Growth rate
 - Initial conditions
 - Reactor type preferences
- The control vector (with U235 fueling of fast reactors permitted) has 6 controls per region plus two shipping controls for fissile from one region to the other
- The state equations (by region) are as shown before and are decoupled except through the control vector (allowing the solver to solve 9 row coupled ODE's rather than 18 row ODE's)
- The adjoint equations (by region) are coupled through the component of the adjoint source arising from the ancillary constraint that ore reserve must be non-negative

The code had been written in MATLAB (using the control theory toolbox)

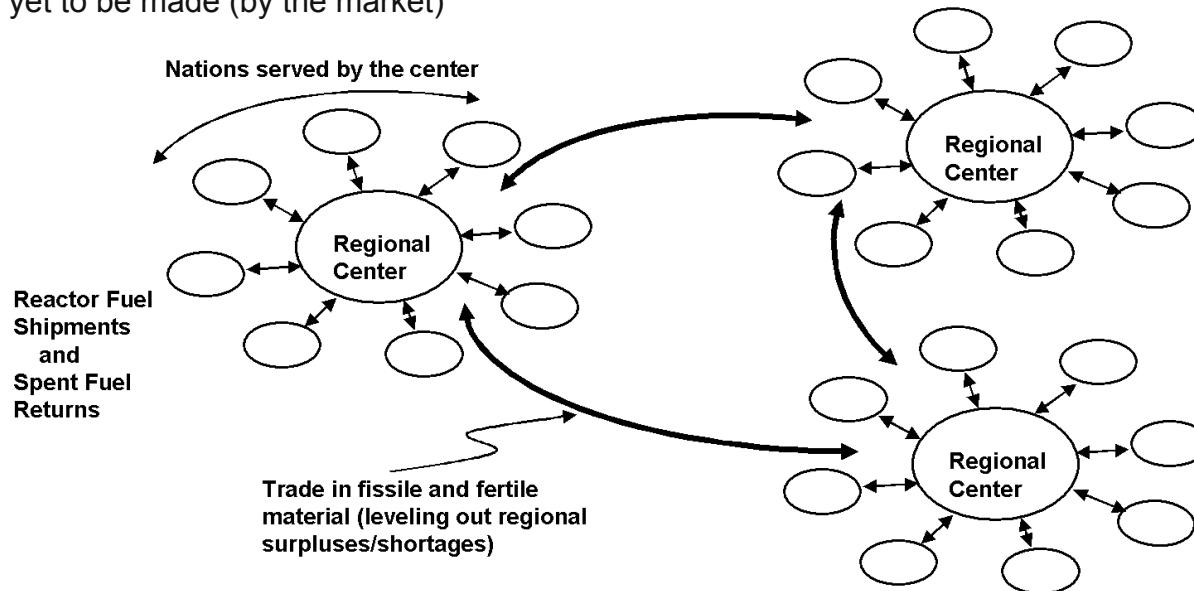
Problem Definition: But No Results Yet

	Region A	Region B	World
Initial Primary Energy (TW_t)			16
World Primary Energy Growth (%/y)			2
Initial World U Ore (k tonnes U)			15.000
Initial Nuclear Energy (GW_t)			
LWR	10^3	30	
FBR	.001	.001	
STAR	.001	.001	
Initial TRU Inventories (tonnes)			
Repro Plant	900	0	
Refab Plant	50	0	
Committed U Ore (k-tonnes U)	?	?	
Reactor Type Preferences (relative)			
LWR	0.5	0.3	
FBR	0.1	0.6	
STAR	0.4	0.1	
Enrichments %			
LWR	4.2	4.2	
FBR	20	20	
STAR	15	15	
Repro/Refab holdup (y)	5	5	
LWR lifetime (y)	35	35	

Conclusions

LWR/Breeder vs (STAR+LWR)/Breeder Strategies

- (1) Both strategies require rapid market penetration rate in the 1st 25 years of the transition
 - Not possible without nuclear deployment in fast growing developing regions
- (2) Both strategies require massive deployments of powerful Breeders
 - Starting immediately
 - Reaching $\geq 50\%$ of the nuclear park before mid century
- (3) Strategy 1 benefits significantly if more ore is found (LWR fraction increases)
 Strategy 2 is less affected by more ore but benefits none-the-less
- (4) The choice of Transition Strategy (LWR/Breeder vs STAR/Breeder) is yet to be made (by the market)
- (5) The Choice Goes beyond Technical Issues:
 - Which one best meets the needs of customers for energy
 - *Is the one that will "sell"*
 - Which one best solves the World's Energy Security/Nonproliferation Dilemma
 - *Is the one governments will seek to enable by institutional measure emplacements*
 - Either strategy could work in the proposed architectures:
 - *Regional Centers (IAEA proposal)*
 - or
 - *Fuel Cycle States/Reactor States (GNEP proposal)*



Cautionary Remarks:

- Models based on classical optimization theory are being used to find constrained transition trajectories to nuclear-fueled sustainable development
- These models are highly idealized
 - Continuous (not discrete) power plant sizes
- These idealizations unphysically accelerate the transition

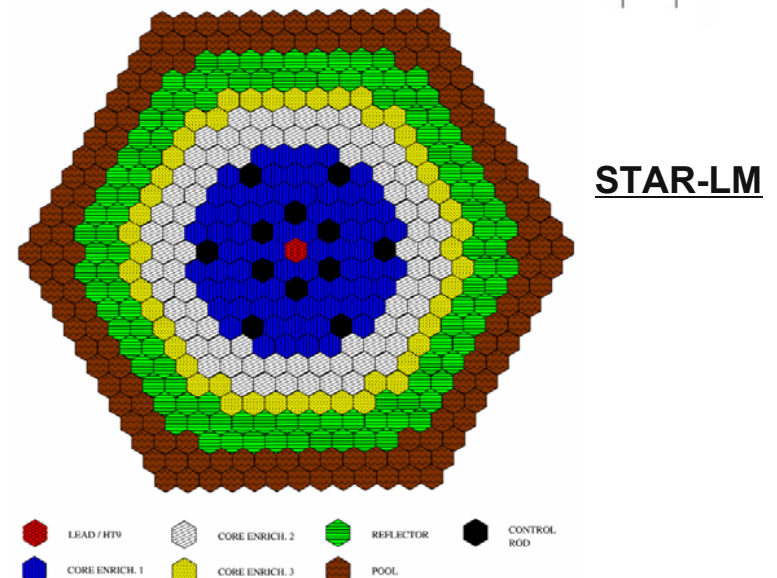
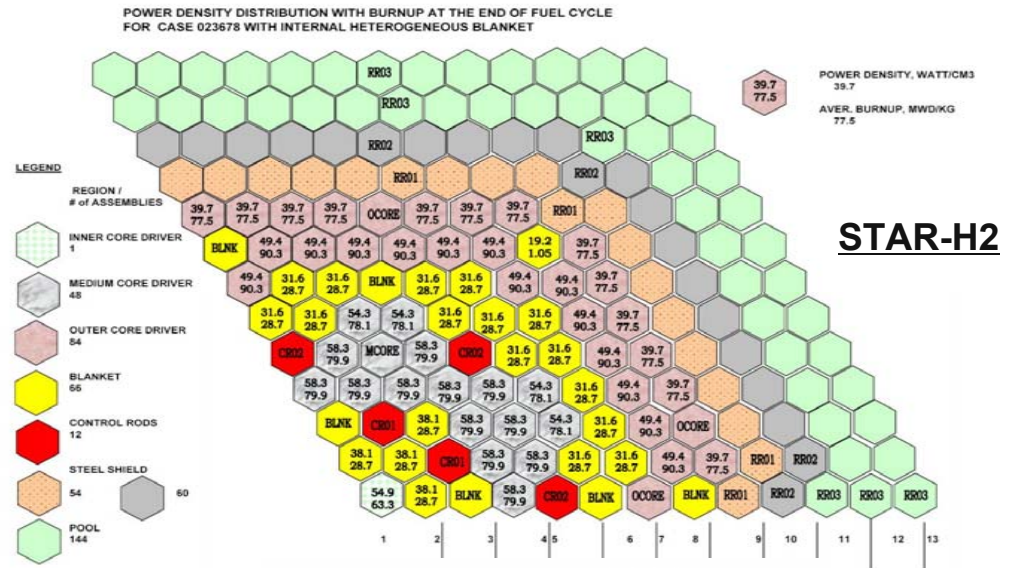
Therefore

- We should take the Strategic Lessons Learned from the idealized model
 - To guide high fidelity dynamic simulations using DANESS, DYMOND, DESAE, etc.
 - Use Multi-regional modeling capability in DANESS (others?) to account for
 - *STAR deployments in developing countries where growth rate is fast*
 - *LWR deployments in developed countries where growth rate is slower but fuel cycle infrastructure is in place*
- We should extend the classes of strategies to include
 - High conversion ratio LWR's
 - AHWR's on a Th/U233 cycle

Backup

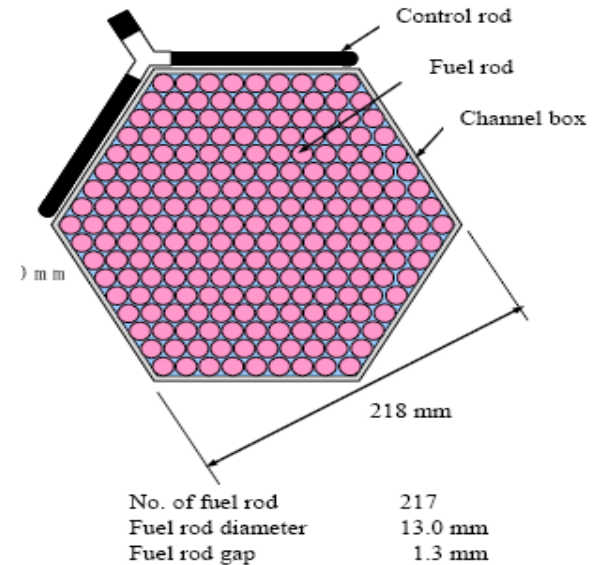
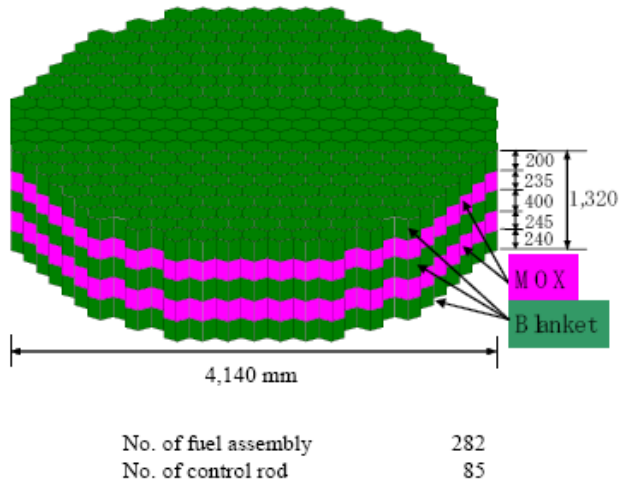
STAR: Small Fast Reactor of 20 Year Refueling Interval

- STAR-H2 and STAR-LM are 400 MW_t, Pb-cooled fast reactors of 15-20y refueling interval designed for a centralized fuel cycle/distributed power reactor energy architecture
- They achieve long refueling interval by CR≅1 and very large incore fissile inventory
- They are documented in the TECDOC: Small Reactors Without Onsite Refueling – to be released in late 2006



RMWR: High Conversion Ratio BWR

- Hitachi, JAERI, JAPC, and TIT have designed a
 - 230 MWe (gross) Reduced Moderation Water Reactor
 - Documented in Annex X of IEAE TECDOC-1485 (March 2006)



- Its Fissile Mass Inventory and Flows are
 - Plutonium fueling
 - Incore Fissile Inventory: $3.368 \text{ kg fissile/GW}_t$
 - Fissile burnup/ $\text{GW}_t \text{ year} = 0.34675$
 - Fissile creation/ $\text{GW}_t \text{ year} = 0.3572$
 - Discharge Burnup $\frac{\text{MW}_t d}{\text{kg fuel}}$: 65 driver; 50 driver + blanket

} CR \square 1

Also:

The Indian advanced design is a AHWR on a cycle where TRU from LWR's & Breeders fuels a Th-converting AHWR

Further Displacing Fossil to ½ Its Current Use by Manufacture of H₂

- Energy planners specify primary energy (i.e., heat)
 - Assuming ~1/3 heat used to make electricity } Assumes heat → electricity @
 - ~1/3 heat used in transportation } η ; 35%
 - ~1/3 heat for process heat } Assumes chemical fuel and 100% heat
- Suppose nuclear delivers its energy in two carriers
 - 1/3 in electricity
 - 2/3 in synthetic chemical fuel based on H₂ manufactured from H₂O at ≲ 50% conversion efficiency
 - to be consistent with planning numbers (53 TW_t by 2100) we have to account for nuclear heat → H₂ manufacture conversion efficiency
- For Fossil Use half of today and synthetic chemical fuel for 2/3 of primary energy:

$$\begin{aligned}
 P_{nuclear}(2100) &= \left(\frac{1}{3} + \frac{2}{3} \frac{1}{0.5} \right) * \left(53 - \frac{1}{2} * 0.9 * 16 \right) \\
 &= \frac{5}{3} 45.8 = 76.3 \text{ TW}_t \text{ y / y}
 \end{aligned}$$

Further Displacing Fossil to ½ Its Current Use by Manufacture of H₂ (Cont'd)

- To Decrease Fossil to half of today's use and to supply energy using electrical and synthetic chemical carriers needed by society

$$\frac{P_{nuclear}(100)}{P_{world}(100)} = \frac{76.3}{53} = 144\% \text{ of world primary energy by year 2100}$$

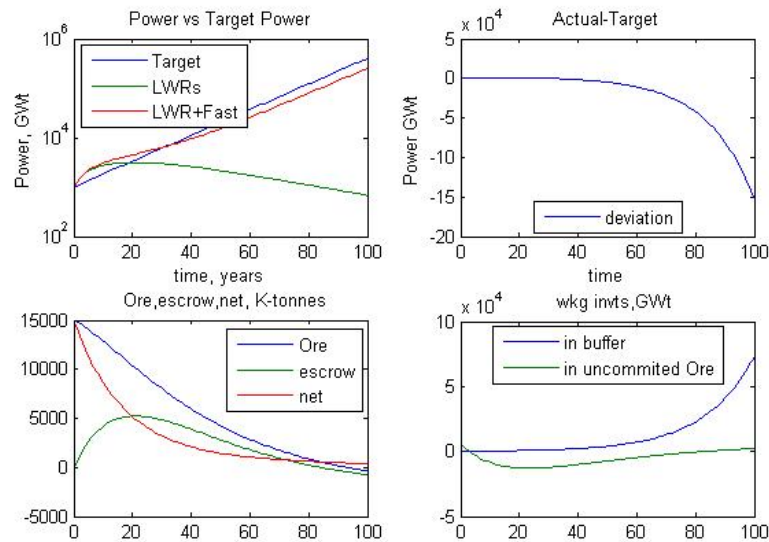
Starting from current world nuclear energy of 1.0 TW_t y/y

$$P_{nuclear}(t) = 1 \text{ g } e^{\alpha_n t}$$

$$\alpha_n = \frac{1}{100} \ln \left\{ \frac{76.3}{1} \right\} = 0.0433$$

- Summary
 - World Primary Energy growth 1.2%/year to 53 TWt y/y by 2100
 - By 2100, to reduce fossil use to half of current level
 - With H₂ manufactured from nuclear heat at η : 50%
 - Nuclear has to grow to ~144% of projected World primary energy (to 76.3 TWty/y (Nuclear growth 4.33%/year)

Upper Bound of Growth (100% Breeder Park) Strategy: LWR → TRU Fueling of Breeders Only ($f=0$)



- Nearly 6%/y growth rate is possible
 - Reaching 250 TW_t y/y by 2100
 - i.e., nearly 500% of world primary energy at year 2100!!

But

- A transition to a 100% Breeder nuclear park:
 - May be undesirable from the interregional equity viewpoint*

Question:

- How can we minimize Breeders and maximize distributed LWR's + STAR's?

In fact, in the early 1970's, Marchetti promoted centralized breeders driving a world hydrogen economy. That architecture for global energy supply would make the regional centers function as synthetic oil fields.

Apply Hamilton's Principle and The Calculus of Variations

To minimize the functional which characterizes of performance over the transition

$$PI = \int_{t_0}^{t_1} f_1(\underline{x}, \underline{u}) dt$$

Subject of constraints

$$f_2(\underline{x}, \underline{u}) \geq 0 \quad t_0 \leq t \leq t_1$$

and

$$f_3(\underline{x}, \underline{u})|_{t=t_1} = 0 \quad ; f_4(\underline{x}, \underline{u})|_{t=t_i} \geq 0$$

We Iteratively, solve a System of Equations

State Equations

$$\frac{d}{dt} \underline{x} = \underline{A} \underline{x}(t) + \underline{B} \underline{u}(t); \quad t_0 \leq t \leq t_1$$

Subject to constraints (eqns. 1, 2, and 3) and to final conditions

$$\underline{x}(t_0) = \underline{x}_0$$

and Euler-Lagrange Equations

$$-\frac{d}{dt} \underline{\lambda} = \underline{A}^T \underline{\lambda}(t) + \underline{B}^* \underline{Ustar}(t)$$

Subject to final conditions

$$\underline{\lambda}(t_1) = \underline{\lambda}_0$$

Where

$$\underline{B}^* \underline{Ustar} = \frac{\delta H_1}{\delta \underline{x}}$$

and

Where the control vector \underline{u} is iteratively improved toward its optimal time-dependent allocations of fissile mass to reactor type by, using a steepest descent algorithm based on:

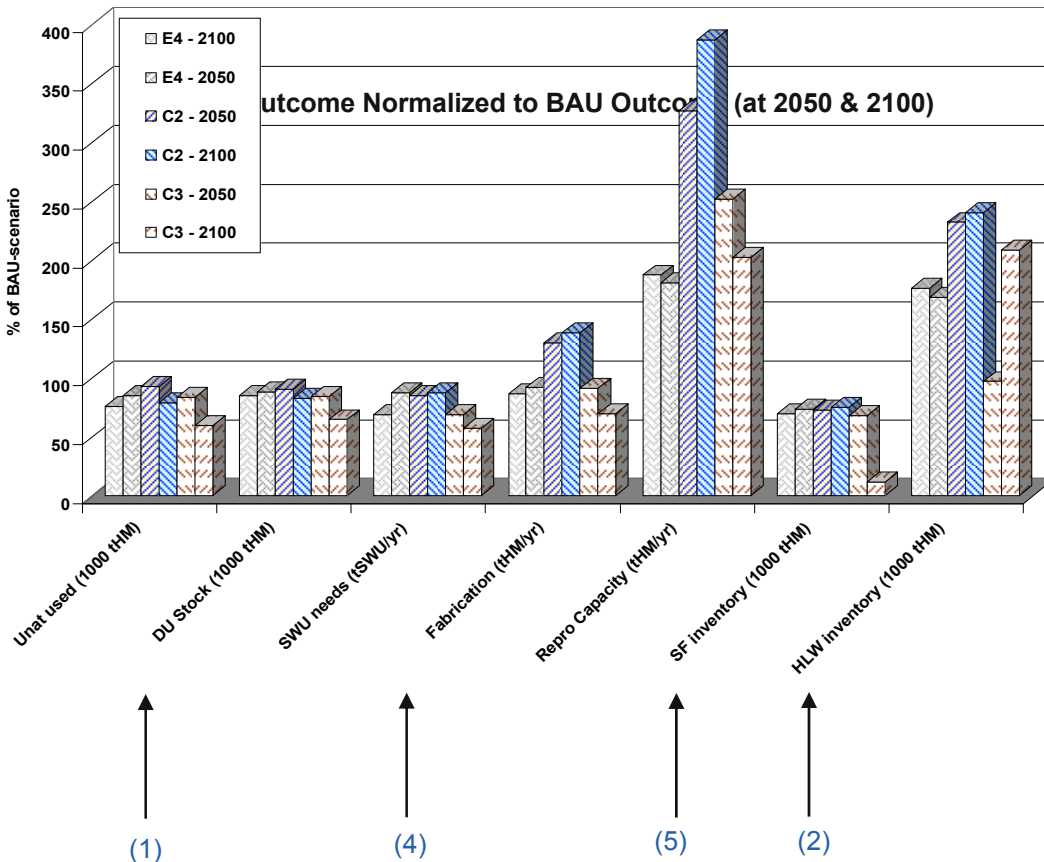
$$\underline{U}_i = \underline{U}_{i-1} - scale * \frac{\delta H}{\delta \underline{u}} \quad (i = iteration\ index)$$

Subject to $\underline{u}(t) \geq$ for all t

Where the Hamiltonian, $H(\underline{x}, \underline{u}, \underline{\lambda})$ is the integrand of the PI plus the sum of Lagrange multipliers times the state and various constraint equations

We use MATLAB and its Control System Toolbox to solve this system of equations

Quantifying World & Regional Payoffs of Mass Flow Symbiosis and Trade Among Regions; Global 05 Paper #567



- Each World region benefits from power plants tailored to their individual needs
 - Developed: Evolutionary Economy of Scale
 - Developing: Innovative Economy of Mass Production

By 2100 for C3

- Less ore withdrawal (because STAR's (CR=1) replace LWR's (CR<<1) in developing regions)
- ~1/6 the SNF inventory (because of symbiotic recycle)
- Major reduction in Pu out of pile (because STAR's store TRU incore compared to LWR MOX store it in SNF)
- ~1/3 Less SWU/year deployed (because TRU-fueled STAR's replace UOX fueled LWR's in developing regions)
- ~ same reprocessing deployed as E4 but achieves reduced SWU and reduced SNF

Some but not all of these Payoff's also will accrue to deployment of thermal reactors of long refueling interval – especially high CR LWR's